A COURSE OF WORKSHOP DRAWING

BY

HENRY A. DARLING, Assoc.M.Inst.C.E.
Formerly Lecturer on Drawing and Geometry at the Municipal Technical Institute, West Ham
Author of "A Course of Elementary Workshop Arithmetic"

AND

F. C. CLARKE, A.R.C.S., B.Sc., A.M.I.E.E.
Principal of the College of Technology, Rotherham

BLACKIE AND SON LIMITED
LONDON AND GLASGOW

By H. A. Darling:

A COURSE OF ELEMENTARY WORKSHOP ARITHMETIC

Second edition 1936.
Reprinted 1942, 1945.
1946

BOOK
PRODUCTION
WAR ECONOMY
STANDARD

THE PAPER AND BINDING OF THIS BOOK
CONFORM TO THE AUTHORIZED ECONOMY
STANDARDS

Printed in Great Britain by Blackie & Son, Ltd., Glasgow

PREFACE

This course of Workshop Drawing has been drawn up by modification and extension of an earlier course of Elementary Workshop Drawing by one of the authors, and is devised to provide a course suitable for all grades of young students in Secondary Schools and Technical Institutes who are taking courses in which a knowledge of technical drawing forms part of the syllabus.

It will be found to cover the requirements, up to and including a Senior first year course, of most technical courses in which drawing is included.

It does not purport to be a course of geometry; wherever expeditious and practical constructions which would be used in modern drawing office practice could be given, these have been followed in preference to rigid geometrical constructions. It definitely discourages the mere copying of finished drawings or examples, and presents problems, examples and exercises in the form and in the language in which they arise in the drawing office of the workshop and factory. Where figures are quoted or data given they may safely be taken as in agreement with actual practice.

It is not suggested that the sequence of the text will be adhered to strictly by students or teachers, as interest in the subject will probably be found to be more readily sustained, and facility in making and reading drawings more speedily acquired, by studying the problems on plane geometry in the earlier chapters concurrently with those on solid geometry in the later chapters.

H. A. D.
F. C. C.

BLACKIE & SON LIMITED
66 Chandos Place, LONDON
17 Stanhope Street, GLASGOW

BLACKIE & SON (INDIA) LIMITED
Warwick House, Fort Street, BOMBAY

BLACKIE & SON (CANADA) LIMITED
TORONTO

CONTENTS

Chap.		Page
I.	Description and Use of Instruments	1
II.	Use of Scales	13
III.	Preparation of Drawings	19
IV.	Lines and Angles	22
V.	The Circle	28
VI.	The Triangle	40
VII.	Plane Figures	47
VIII.	Regular Polygons	53
IX.	Mensuration of the Circle	56
X.	Simple Solids—Developments	60
XI.	Sketching	65
XII.	Pictorial Projection	67
XIII.	Isometric Projection	73
XIV.	Orthographic Projection	79
XV.	Sectional Views	87
XVI.	Intersections of Solids	98
XVII.	Developments of Surfaces	107
XVIII.	Helices and Screw Threads	113
XIX.	Loci	117
XX.	Lines and Planes	122
	Miscellaneous Exercises	128
	Numbers, Squares, Cubes, Square Roots and Cube Roots	134
	Diameters of Circles, Circumferences and Areas	137

WORKSHOP DRAWING

CHAPTER I

DESCRIPTION AND USE OF INSTRUMENTS

In the choice of drawing instruments a wide range of quality and price presents itself, and hard-and-fast rules cannot be laid down, but students will appreciate that the work produced inevitably suffers if the tools and implements employed are poor in quality. For the use of beginners many makers provide at small cost "school sets" in which the pieces are few in number but quite suitable for elementary work; but where convenient it is advisable that instruments should be procured singly, as circumstances permit or as need arises, and of such quality as will enable them still to be used at a later period when the drawing undertaken by the student is of a more exacting character.

The following is a list of the instruments and appliances which will be found necessary in working through this course, and some brief notes are given to assist in their selection.

Drawing Board, 24 in. long, 17 in. wide, of pine, battened at the back to prevent warping.

Paper, "half imperial" size, 22 in. by 15 in. Cartridge paper will be found quite suitable.

Drawing Pins.

T Square, with "blade" 24 in. long; the blade should not be let in flush with the "stock" or cross piece.

Two Set Squares.—One of 45° with each of the two edges containing the right angle 6 in. long.

One of 60°, with the longer of the two edges containing the right angle 8 in. long.

These may be obtained, at small cost, in pear wood, but preferably should be of transparent celluloid, about $\frac{1}{16}$ in. thick and not bevelled at the edges.

A Pair of Pencil Compasses, with legs say $4\frac{1}{2}$ or 5 in. long. The pivot leg should have a round point, preferably a needle point. Cheap compasses with triangular points should be studiously avoided.

A Pair of Dividers, with legs say $4\frac{1}{2}$ in. long.

A Pair of Spring Bows for Pencil.—These are for describing circles of small radius, but while a decided convenience are not indispensable for the work of elementary students.

Scales.—Convenient sets of cardboard scales are to be had at small cost, but preferably two boxwood scales should be obtained, one a 12-in. oval-section engineers' scale, " open-divided ", containing on one side scales of $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, and 1 in. to 1 ft., and on the other $\frac{3}{8}$, $\frac{3}{4}$, $1\frac{1}{2}$, and 3 in. to 1 ft.; and the other a scale on which inches are divided decimally, and centimetres " close-divided " to read to millimetres are given.

Protractor.—Semicircular in form, of transparent celluloid or horn, about 5 in. diameter, divided to read to half-degrees. Rectangular protractors are not recommended.

Pencils.—One moderately hard, say H or HH, and one softer, say HB.

Pricker.—This is used for pricking through diagrams from one paper to another, and also for marking off dimensions from the scale to paper. A convenient form can be made by inserting a portion of a fine needle in a piece cut from a penholder, and paring away the wood at the point.

Piece of Soft Indiarubber.

In addition to the foregoing students should also be provided with the following articles for measuring up and sketching objects preparatory to making scale drawings of them.

A Fourfold Boxwood 2-ft. Rule.

An Engineers' 12-in. Steel Rule, of Chesterman's or other suitable make.

Calipers.—One pair each of inside and outside calipers.

Use of Instruments

Pencils.—Of the two pencils the harder is to be used for ruled lines. It should be sharpened to a chisel point, and in working should be held in an almost upright position, i.e. nearly square with the surface of the paper, with the flat side of the point in close contact with the edge of the set square or T square.

In drawing lines with the T square in the conventional horizontal position, i.e. those at right angles to the working edge of the drawing board, the pencil should be moved from left to right. Lines at right angles to these should be drawn with the set square placed with an edge containing the right angle on the edge of the T square, the pencil being moved from bottom to top when on the left-hand side of the set square, and from top to bottom when on the right-hand side.

The softer pencil is the writing pencil and is to be used for lettering and dimensioning on the drawing. It should be sharpened to a round or conical point.

The pencil in the compasses and spring bows should be in each case sharpened to a chisel point, and should be of the same degree of hardness

as the drawing pencil. To ensure this use a piece of lead cut from a scrap of the drawing pencil. A small piece of fine emery cloth or glasspaper, or a small " dead smooth " file, should be kept at hand for maintaining the keenness of the pencil points, the penknife only being used for paring away the wood.

Board and T Square.—The drawing board will almost invariably be used with the longer edges placed from left to right in front of the worker. The essential feature of the board is that one edge should be truly straight. This is the short edge on the left-hand side, called the " working " edge. The angles at the corners of the board, although generally made so, need not necessarily be right angles, and it must not be assumed that they are. The T square, for this reason, should only be used for setting out lines at right angles to the working edge, and not applied to any other edge of the board, as if this is done any inaccuracy in the angles of the drawing board will be reproduced in the drawing. The stock of the square must be kept closely against the working edge of the board, and the square shifted by means of the stock held in the left hand; the blade must not be used at all in shifting the square. The T square is used in drawing those lines only which run from left to right on the paper—commonly referred to as horizontal lines—perpendiculars to these being drawn by the use of the set squares.

Dividers.—These will be generally used for transferring distances from the rule or scale to the paper or from one part of the drawing to another, for subdividing lengths by trial, and for " stepping out " multiples of any given distance on the drawing. The head joint should be tight enough to allow of opening and closing easily but without liability to slip.

Protractor.—This is used for setting out and measuring angles, and the form recommended is shown in fig. 1. The unit for the measurement of angles is the degree (°) and is obtained as follows:

If the circumference of any circle is divided into 360 equal parts, and the points of division joined to the centre of the circle, the angle included between any two consecutive radii is equal to one degree. The value of the angle thus formed is independent of the radius of the circle. A right angle contains 90°, and is swept out by a line which revolves about one end as a centre through a quarter of a complete revolution.

To measure a given angle with the protractor; for example, the angle ABC, fig. 2:

Place the protractor over the angle to be measured, with its centre at B and the base line on BC, and read off the division where the line AB (continued if necessary) crosses the edge of the protractor, taking care to refer to the set of numbers which commence at zero at C. The common

form of protractor illustrated is numbered from both ends of the base line, to permit of angles being read from either side of the centre. Thus the angle ABC will be found to contain $44\frac{1}{2}°$.

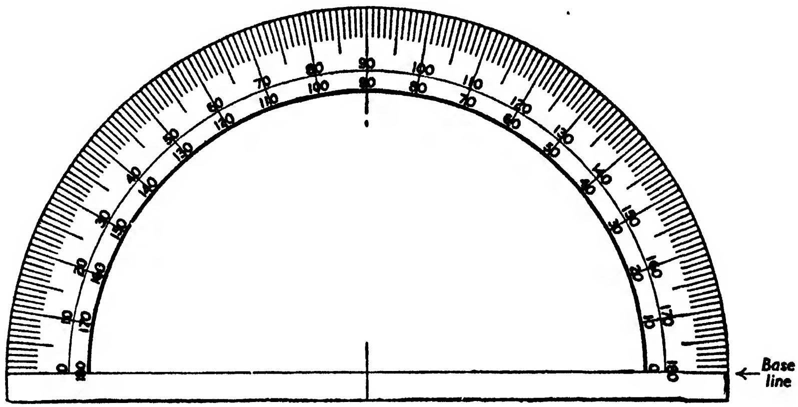


Fig. 1

To set out any required angle with the protractor; for example, from the point D on the line EF (fig. 3) to set out a line making an angle of 52° with EF:

Place the protractor with the centre at D and the base line on EF, and at the point of division on the edge of the protractor corresponding with

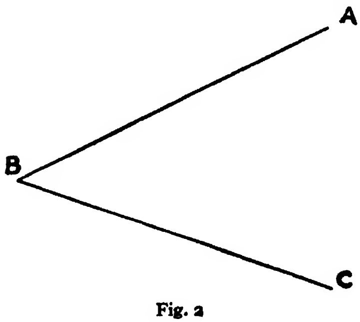


Fig. 2

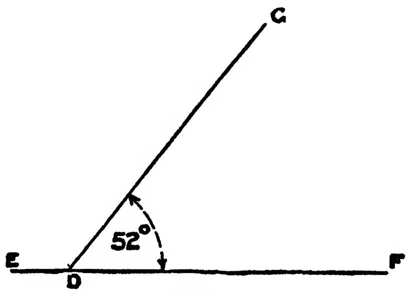


Fig. 3

52° make a mark as at G, with a sharp pencil (or with a pricker). Remove the protractor and join this mark to D. The angle GDF will then contain 52°.

For practical purposes angles are generally stated in degrees and decimal fractions thereof, and with careful work, using the size of protractor suggested, angles may be readily set out or determined to within ·2 of a degree.

Tests and Adjustments of Board, T Square, and Set Squares

Drawing Board.—As before stated, the working edge of the board must be straight. This may be tested by trying it against a standard straightedge, any necessary correction being made at a carpenter's bench by truing up the edge with a shooting plane. It is convenient, but not essential, that the other edges also should be straight, so that, if occasion demands, another edge may be used as a working edge. Since the T square will never be used on more than one edge for any particular drawing, the corners need not be square. They are generally made so, however, and may be tested by means of a joiner's square.

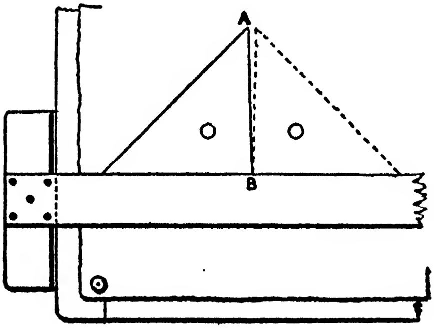


Fig. 4

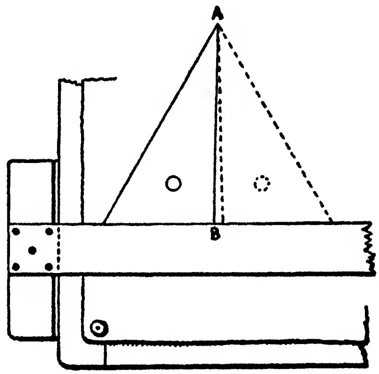


Fig. 5

T Square.—The working edges of the stock and blade must be straight. To test the straightness of the blade, rule a fine line with a hard pencil the full length of the paper. Turn the square over (but not end for end) and apply the same edge to this line. If the edge coincides therewith throughout the length, the blade is straight; if not, it must be detached from the stock and shot true with the plane. If the blade has a bevelled edge it may be tested against a standard straightedge. The stock should be tested in the same way.

Set Squares.—First test the right angle as follows:

Place the set square in position on the edge of the T square, as shown in figs. 4 and 5, and with a hard pencil draw a fine line the full length of the vertical edge AB. Reverse the set square into the position indicated by dotted lines, and apply it to the line already drawn. If the edge, when reversed, coincides with the line, the angle is a right angle. If, however,

there is any divergence, draw a second line. This will make a small angle with the first, and the angle of the square must be corrected by an amount equal to half the angle of divergence between the two lines drawn.

To Test the Angles of 45°.—When the square has been found to be right angled or has been made so, the angles of 45° may be tested as follows:

Place the set square in position on the edge of the T square, as shown in fig. 6, and draw a line along the edge AC. Then place the edge CB along this line and draw another line CA′. If the whole angle A′CB now agrees with the right angle CBA, the three angles of the set square are correct. If not, the angle at C must be corrected by planing the edge CA by an

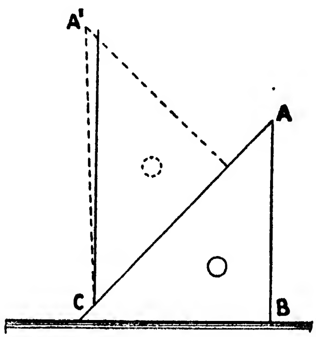


Fig. 6

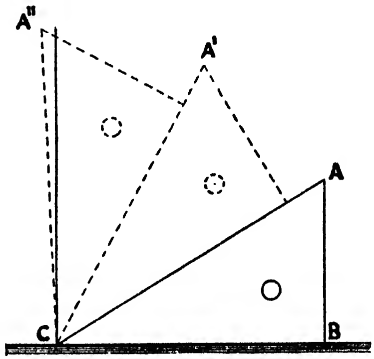


Fig. 7

amount equal to half the difference between the angle A′CB and the right angle.

To Test the Angles of 30° and 60°.—The right angle having been tested, and corrected if necessary, the angle of 30° may be tested as follows:

Proceed as given above for the angle of 45°, but in this case apply the angle at C three times, as indicated in fig. 7. If upon trial the whole angle A″CB now agrees with the right angle, all three angles of the set square are correct. If not, the angle at C must be corrected by planing the edge CA by an amount equal to one-third the difference between the angle A″CB and the right angle.

In adjusting the values of the angles the correction is best made by planing with a steel-faced plane. This is an operation of some delicacy, necessitating expert manipulation of the plane. If the student is not thoroughly expert in its use, he will be well advised to entrust the operation to someone who is.

EXERCISES

1. Add together the three angles of a 45° set square; do the same for a 60° set square. Draw any triangle, and add the three angles. Do this for a number of cases, and verify that: **The three angles of any triangle are together equal to two right angles.**

If a triangle has one of its angles a right angle, it follows, therefore, that the sum of the other two must also be a right angle.

2. Given a reputed 45° set square, if the two lesser angles are found to be equal does it follow that the remaining angle is a right angle?

3. A right-angled triangle (i.e. one in which one angle is 90°) has one angle 35°; set out and measure the value of the remaining angle.

4. Two angles of a triangle are found to measure 23° and 75° respectively. What would the third angle measure?

5. Measure the sides of your 45° and 60° set squares. Notice that the longest side is opposite the greatest angle. Draw a number of triangles at random and verify that: **In any triangle the greatest side is opposite to the greatest angle.**

Notice that the two short sides of the 45° set square are equal, i.e. the sides opposite the equal angles are equal. Measure the perpendicular distance from the right-angle corner to the opposite side, and note that this is equal to half the longest side, and that it divides the triangle into two equal and similar parts. Notice that two equal 45° set squares placed with their longest edges together form a square; that is, the shape of the 45° set square is formed by cutting a square in halves along a diagonal as in fig. 8.

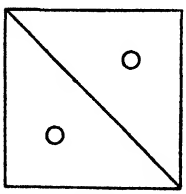


Fig. 8

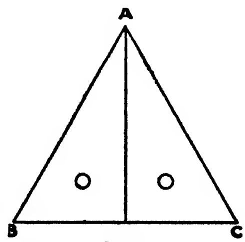


Fig. 9

Measure the sides of your 60° set square. Notice that the shortest side is half the length of the longest. Measure the perpendicular from the right-angle corner to the opposite side, and note that it is half the length of the side opposite the angle of 60° and divides the triangle into two smaller 60° triangles. Note that two equal 60° set squares placed together, as in fig. 9, form a triangle ABC in which all angles are equal and all sides of the same length, i.e. an "equilateral triangle", so that the 60° set square is half an equilateral triangle.

It is highly important that the properties of the set squares should be

well studied and their use and manipulation thoroughly practised. The beginner, who starts probably with the idea that they are mainly of service as straightedges, or for drawing lines at 30°, 45°, 60°, and 90° with the T square edge, will then find that they have a much more extended application, and will find the time spent in mastering their use well repaid in speed and accuracy of working.

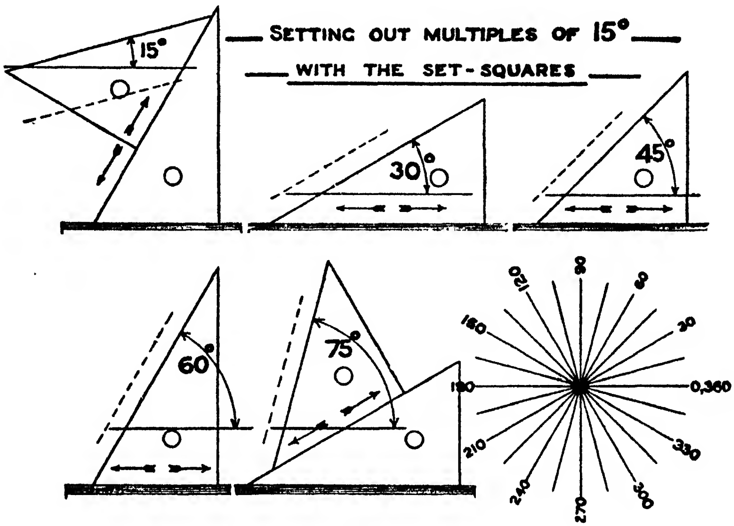


Fig. 10

By shifting the set square shown in the direction indicated by the arrows, any number of lines parallel to the given direction may be drawn (see dotted line in each case).

Fig. 10 illustrates how any multiples of 15° may be set out by a combined use of the two set squares.

EXERCISES

6. Draw six circles, each $2\frac{1}{2}$ in. diameter, and in them respectively inscribe, using the set squares only, regular figures having the following numbers of sides:

(*a*) 3, i.e. an equilateral triangle; (*b*) 4, i.e. a square; (*c*) 6, i.e. a regular hexagon; (*d*) 8, i.e. a regular octagon; (*e*) 12; (*f*) 24.

About the same six circles describe, using the set squares only, regular figures having 3, 4, 6, 8, 12, and 24 sides respectively.

Note.—A " regular " figure is one having all its sides equal in length, and the angles between adjacent sides equal.

Figs. 11 to 16, which may be drawn using the set squares only, represent

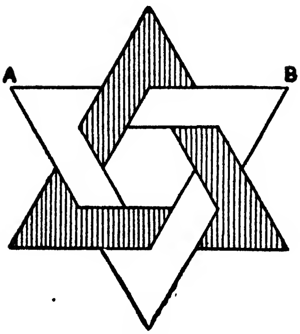


Fig. 11

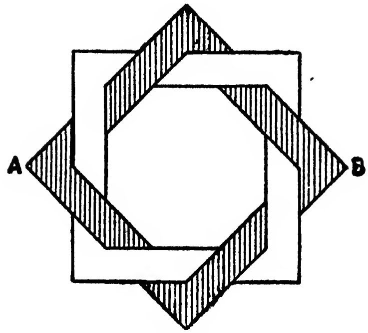


Fig. 12

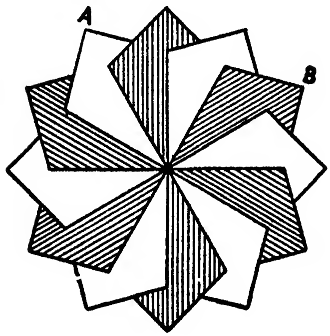


Fig. 13

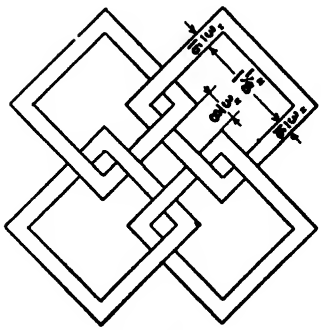


Fig. 14

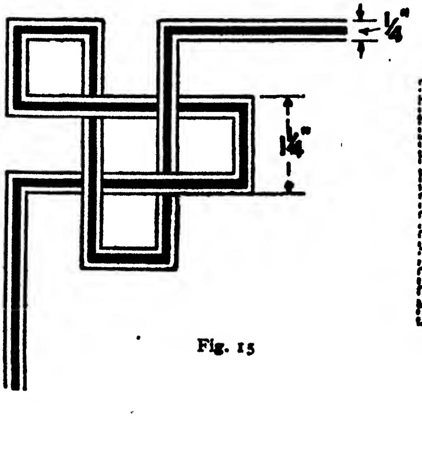


Fig. 15

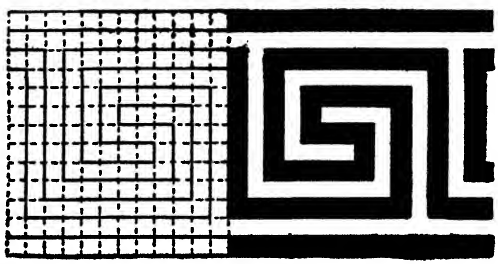


Fig. 16

geometrical patterns based upon simple figures, and are suitable for inlaying, marquetry, tiling, &c.

7. Draw the interlaced triangles shown in fig. 11, making the length AB 3 in.

8. Draw the interlaced squares shown in fig. 12, making the length of diagonal AB $2\frac{3}{4}$ in.

9. Draw the pattern in fig. 13, formed of three squares placed as shown, making the side of a square such as AB 2 in. in length.

10. Draw the interlacing squares in fig. 14.

11. Fig. 15 shows a corner suitable for the border of a drawing or an inlay edging. Set this out to the dimensions given.

12. Fig. 16 shows a portion of a continuous band pattern of the form known as the Greek Key, and on the left the method of setting out on small squares. Draw a portion of the pattern, making the side of the small squares $\frac{3}{16}$ in.

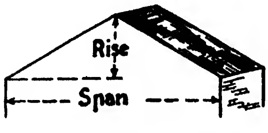


Fig. 17

Fig. 18

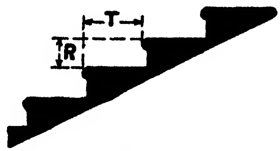


Fig. 19

13. The "ramp" at the end of a railway station platform slopes 1 in 8, i.e. there is a rise of 1 ft. vertically for every 8 ft. measured horizontally. Measure the angle of slope in degrees.

14. The angle of slope of a pitch roof is generally fixed by giving the ratio of rise to span (fig. 17). Set out the slope for roofs having rises of $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{3}$ the span respectively, and measure and write down the angle in degrees in each case.

15. A wall is built to a "batter" of 1 in 10, i.e. for every 10 ft. of its height measured vertically the face recedes 1 ft. measured horizontally. What angle does the face make with the horizontal, and what angles correspond with batters of 1 in 8 and 1 in 12 respectively?

16. On a flight of stairs, if a straightedge were placed to touch the front edges or nosings of all the stairs, the angle which this makes with the horizontal is termed the "angle of going" of the stair (fig. 18). Measure the angle of going for a stair in which the "risers" are 6 in. high and the "treads" 11 in. wide.

17. In fig. 19, if the tread T is 12 in. and the riser R $5\frac{1}{2}$ in., what is the angle which the handrail of such a stair would make with the horizontal?

18. Draw the patterns shown in figs. 20 and 21, making the outside

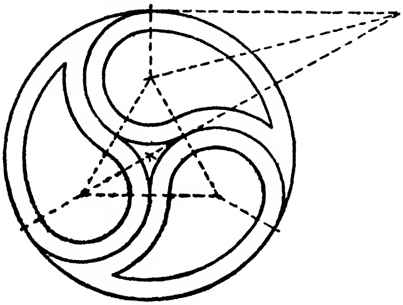


Fig. 20

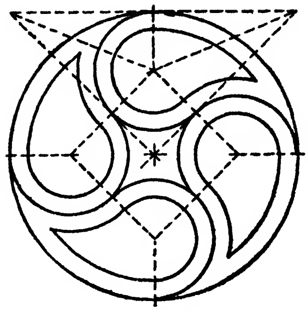


Fig. 21

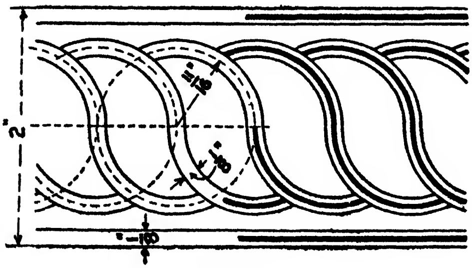


Fig. 22

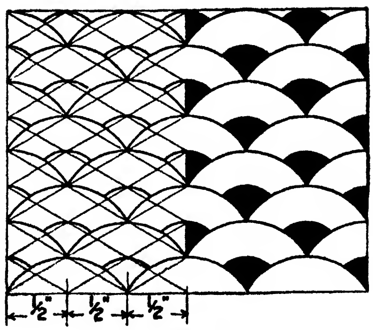


Fig. 23

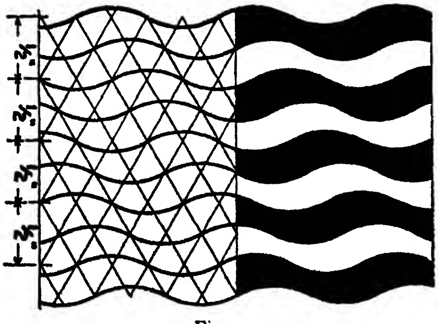


Fig. 24

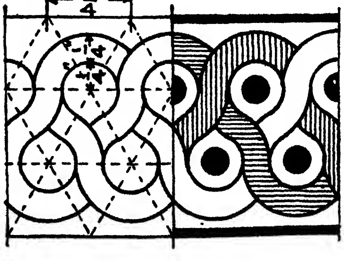


Fig. 25

diameter 3 in. and the width of band $\frac{3}{16}$ in. The method of arriving at the centres of the circular arcs is indicated by dotted lines.

19. Draw a portion of the continuous band pattern for an inlay shown in fig. 22 to the sizes given.

20. Draw portions of the "all over" repeated pattern (fig. 23), the "wave" pattern (fig. 24), and the interlaced bands (fig. 25) to the sizes given.

CHAPTER II

USE OF SCALES

In preparing drawings it is impracticable beyond certain limits in the size of the objects represented to draw them full size, and it becomes necessary to draw to a reduced size when dealing with large objects. So long as the degree of reduction is the same for all parts of the object, we obtain a true and properly proportioned representation on the drawing—as it were in miniature—and the drawing will be like the object, but on a smaller scale. The degree of reduction must be arranged to suit the circumstances, for it will be apparent that the scale of reduction suitable for drawing a piece of machinery would not serve for representing the factory in which it is housed, nor would the scale in this case serve for representing a plan of the parish in which the building is situated. The choice of a suitable scale is governed mainly by two considerations: it must be small enough

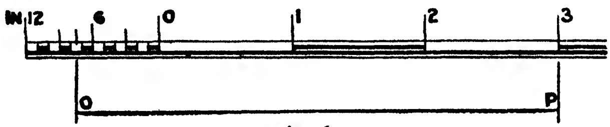


Fig. 26

to admit of the drawing being made on a sheet of convenient size, while large enough to ensure that all necessary details are clear and readable.

The scales most generally in use by engineers and architects are 3 in., $1\frac{1}{2}$ in., $\frac{3}{4}$ in., $\frac{3}{8}$ in., 1 in., $\frac{1}{2}$ in., $\frac{1}{4}$ in., and $\frac{1}{8}$ in. respectively to 1 ft.

Surveys and key plans may be made to scales of one mile to 1, 4, or 6 in., 100 ft. to 1 in., 4 chains to the inch, and so on.

In trades where the parts manufactured are very small, for instance, in mathematical instrument making, watchmaking, &c., drawings are frequently made to scales of enlargement, i.e. twice full size, ten times full size, &c., as may be suitable.

Fig. 26 shows part of a scale divided to read feet and inches. Fractions of an inch on such a scale are to be estimated by eye.

The numbering of the scale is important. Zero mark is placed at the

commencement of the undivided intervals, inches being numbered therefrom towards the left, and feet towards the right as shown. When adding a scale to a drawing, it is usual to finish it off by filling in alternate divisions with a heavy line (the foot space 0, 1, being open), and to underline it with two lines, one heavy and the other light.

To Measure a Dimension with the Scale.—For example, to set out a dimension of 2 ft. 3 in. Mark the point numbered 2 on the undivided feet, and the point representing 3 on the divided foot. The distance between these points represents 2 ft. 3 in. The line OP (fig. 26) represents a length of 3 ft. $7\frac{1}{2}$ in.

In measuring with boxwood or cardboard scales, the scale edge should be applied direct to the paper, and the dimension marked off thereon with a sharp pencil or with a pricker. Transferring dimensions with the dividers should be avoided, as it is apt to lead to inaccuracy and to deface the scale.

No drawing or exercise can be considered complete on which the scale is not stated, and on important drawings the scale is drawn at the foot, so that by its aid missing dimensions may be supplied, and also in order that when the paper expands or contracts, as it will according to the state of the atmosphere, the scale shall vary with it to the same extent.

Representative Fractions of Scales.—Scales may be stated by giving the denomination of the scale, e.g. $1\frac{1}{2}$ in. = 1 ft. 0 in., or by giving the fraction of full size to which dimensions are reduced, i.e. $\frac{1}{8}$ full size. Or if 1 in. represents 1 ft. 0 in., dimensions are reduced in the ratio 1 : 12, and the drawing will be to a scale of $\frac{1}{12}$ full size.

These fractions are termed the representative fractions of the scales.

Denomination of Scale	Representative Fraction	Denomination of Scale	Representative Fraction
3 in. = 1 ft.	$\frac{1}{4}$	1 in. = 1 ft.	$\frac{1}{12}$
$1\frac{1}{2}$,, = 1 ,,	$\frac{1}{8}$	$\frac{1}{2}$,, = 1 ,,	$\frac{1}{24}$
$\frac{3}{4}$,, = 1 ,,	$\frac{1}{16}$	$\frac{1}{4}$,, = 1 ,,	$\frac{1}{48}$
$\frac{3}{8}$,, = 1 ,,	$\frac{1}{32}$	$\frac{1}{8}$,, = 1 ,,	$\frac{1}{96}$

Where the metric system is in use decimal scales are most commonly employed, such as $\frac{1}{10}$, $\frac{1}{20}$, $\frac{1}{50}$, $\frac{1}{100}$ full size. These are stated thus: $\frac{1}{10}$ full size, or scale 1 : 10.

The construction of scales in the metric system is much more simple than is the case with those constructed for our British units, as will be seen from fig. 27, in which the full-size scale of centimetres divided to read to millimetres is given, and below it scales of 1 : 10 and 1 : 100 respec-

tively. It will be seen that the divisions are identical, and that it has only been necessary to modify either the numeration or the nomenclature to produce the scales of $\frac{1}{10}$ and $\frac{1}{100}$.

In workshops and drawing offices where drawings are duplicated by photographic processes in which prints are taken from tracings used as

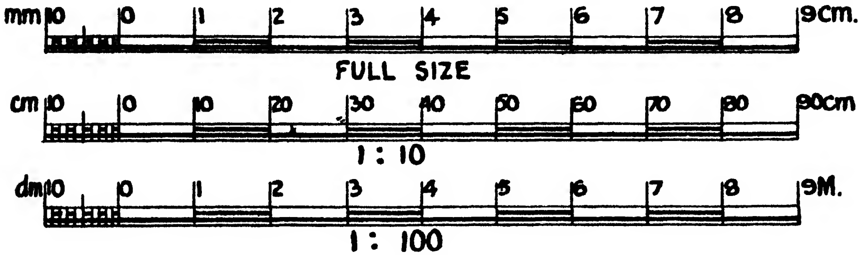


Fig. 27

negatives, the "photo prints" so obtained seldom scale accurately. This is due to the fact that in developing they have to be soaked in baths of liquid, and when dried generally shrink. Where the scale has not been drawn on the tracing it is often necessary in practice, therefore, to construct an adjusted, or as it is termed a "shrunk" scale, to enable dimensions to be correctly measured from such prints.

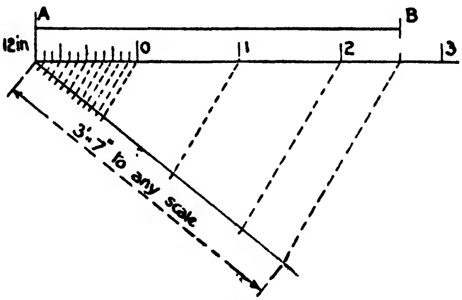


Fig. 28

This is done by choosing any known dimension and constructing the scale by the method given below.

Example.—**The given line AB represents a length of 3 ft. 7 in. To construct the scale to which it is set out** (fig. 28). Draw a line, and on it set out AB. From one end draw a line making any angle therewith, and on it mark out to any suitable scale a dimension of 3 ft. 7 in., as shown, marking the subdivisions on the first foot. Join the ex-

tremities, and draw parallels through the points of division to cut the length AB. This will give the required scale, which can then be extended in length as required.

A practical application of an adjusted scale occurs in the patternmaker's rule. Castings are made by running molten metal into moulds in sand or other suitable material, the cavity in the mould being formed by the use of a wooden pattern of the necessary shape, which is withdrawn after the sand has been tightly packed round it. Due to the contraction of the metal in cooling, the castings when taken out of the moulds are smaller than the patterns from which they are moulded. This contraction, therefore, must be allowed for, and in making the pattern a rule is used on which the divisions are larger by the requisite amount than those on an ordinary 2-ft. rule. The extent of the contraction varies according to the metal of the casting. It will be seen that the patternmaker's rule is really an expanded scale.

Where large numbers of similar castings are required, it is frequently found that a wooden pattern is not sufficiently durable to stand the wear and tear of repeated use. In such a case a metal pattern may be prepared. This is done by making a wooden pattern first, and casting a metal pattern therefrom, which is used for making the actual castings. The patternmaker, therefore, has to allow for two contractions, one occurring when the metal pattern is cast from the wooden one, and another when an actual casting is made from the metal pattern. This is termed allowing " double contraction ".

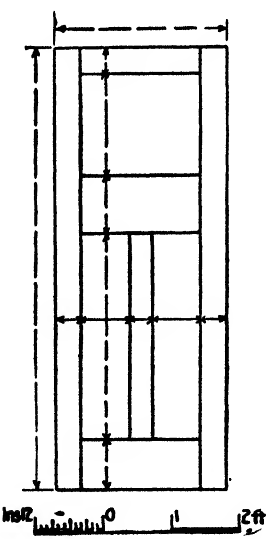


Fig. 29

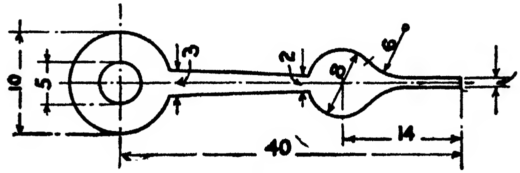


Fig. 30

EXERCISES

1. Fig. 29 shows a drawing of a 3-panel door, and also gives the scale to which it is drawn. Fill in the dimensions by the use of the given scale.

2. A workman is given a detail drawing made to a scale of 1 in. to 1 ft. 0 in., and wishes to supply a missing dimension. He applies in error a scale of $1\frac{1}{2}$ in. to 1 ft. 0 in., and so obtains an incorrect dimension of 2 ft. 4 in. What is the correct dimension?

3. Give the representative fractions of the following scales:
2 in. = 1 ft. 0 in., $1\frac{1}{4}$ in. = 1 ft. 0 in., 20 ft. to 1 in., 1 chain to 1 in., 6 in. to 1 mile.

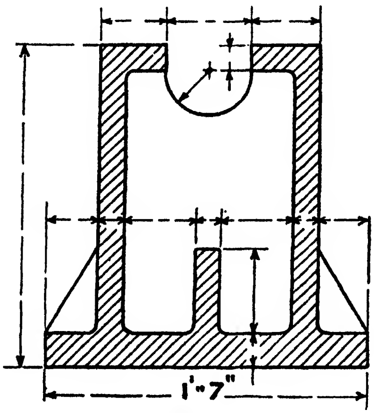


Fig. 31

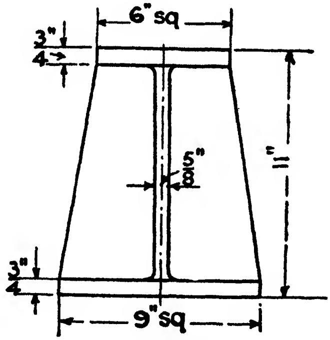


Fig. 32

4. Fig. 30 represents the hand of a watch, dimensions being given in hundredths of an inch. Draw this to a scale of 10 times full size.

5. Fig. 31 is drawn to scale. Construct the scale to which it is made, use it to supply the missing dimensions, and redraw to $1\frac{1}{2}$ in. to 1 ft.

6. Cast iron contracts in cooling $\frac{1}{8}$ in. for each foot of the finished linear dimensions, i.e. where the pattern measures $12\frac{1}{8}$ in. the casting will measure 12 in. Construct a patternmaker's rule suitable for use in making patterns for iron castings.

7. By the use of the patternmaker's rule made as above set out full size the pattern which would produce the casting shown in fig. 32.

8. Construct a patternmaker's rule suitable for making patterns for brass castings, having given that the contraction for brass is $\frac{3}{16}$ in. per foot.

9. By the use of the above rule set out full size the pattern for the casting shown in fig. 33.

10. Fig. 34 represents an iron casting. A large number of these being

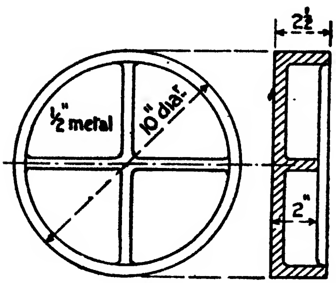


Fig. 33

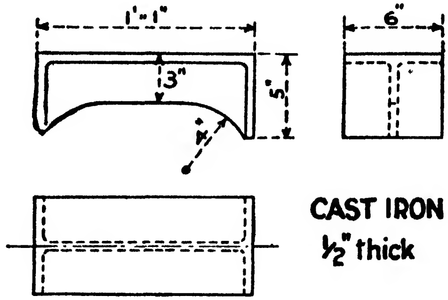


Fig. 34

required, it is desired to prepare a brass pattern. Draw half full size the wooden pattern from which such a brass pattern would be cast, allowing " double contraction " (i.e. $\frac{5}{16}$ in. per foot).

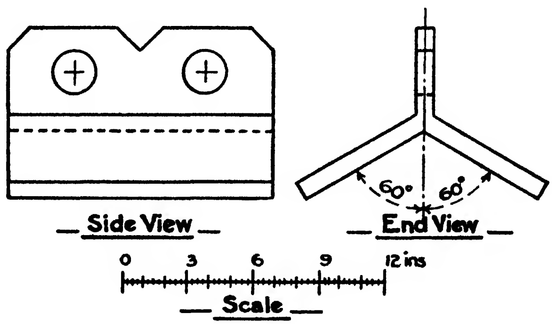


Fig. 35

11. Fig. 35 gives two views of a terra-cotta ridge tile, together with the scale to which they are drawn. Draw the views to a scale of 3 in. to 1 ft., obtaining all necessary dimensions by the use of the scale given with the figure, and writing them on your drawing.

CHAPTER III

PREPARATION OF DRAWINGS

It is assumed that throughout the course of drawing laid down in this book all exercises will be completed in pencil. The preparation of finished inked-in drawings on paper is rarely called for under modern conditions of workshop and drawing-office practice, and should not be attempted in class until facility has been acquired in producing neat and accurate pencil work.

The recommendations contained in British Standard Specification No. 308 on Engineering Drawing Office Practice should be followed wherever possible.

Lines.—The **principal lines** or outlines of the drawing should be firm continuous lines, or, as they are termed, full lines all of the same " strength ". A firm line does not necessarily mean a thick one, the first consideration being that the lines should be readily distinguishable, but they must not be so thick as to affect the value of any dimensions which have to be determined by measurement from the drawing. In solving problems from which results are to be obtained by measuring, it is important that the lines should, for the sake of accuracy, be as fine as is consistent with legibility; on a dimensioned detail drawing, however, a thicker line is permissible and indeed desirable.

Dotted Lines may be used to distinguish any subordinate portion from the main portion of a diagram, or, as will be seen later, to represent on a projection parts which are hidden from view. These should be of the same strength as the full lines, and consist of a series of approximately equal short lines separated by spaces—each stroke properly finished. Slovenly dotted lines, such as shown at F (fig. 36), will ruin the appearance of any drawing.

Centre Lines should be thin lines, " chain dotted ", i.e. consisting of alternate long and short strokes, each properly finished, as at C (fig. 36).

Dimension Lines should be fine continuous lines interrupted to admit the figures of the dimension. Where an exercise requires that construction lines be left in to make clear the method employed in solving the problem, these may be of the same strength as the dimension lines.

Arrowheads.—These are placed at the extremities of the dimension lines to indicate the points between which the dimension is taken. They must be neatly formed, extend exactly to the ends of the dimension lines they limit, and be connected to the part to which they refer by short cross

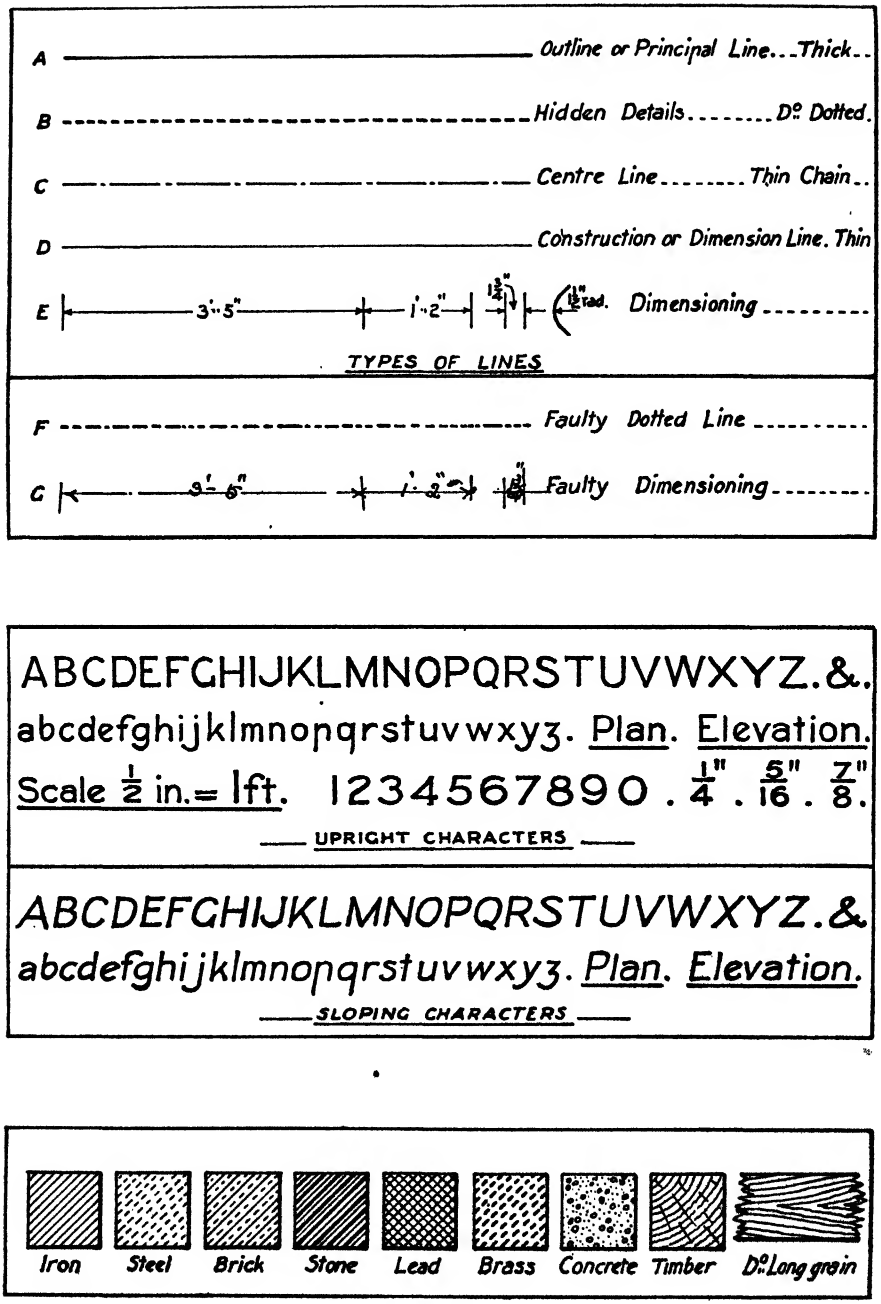

A Outline or Principal Line...Thick..
B Hidden Details........D° Dotted.
C Centre Line........Thin Chain..
D Construction or Dimension Line. Thin
E 3'-5" 1'-2" Dimensioning........
TYPES OF LINES
F Faulty Dotted Line..........
G Faulty Dimensioning........
ABCDEFGHIJKLMNOPQRSTUVWXYZ.&.
abcdefghijklmnopqrstuvwxyz. Plan. Elevation.
Scale ½ in.= 1ft. 1234567890 . ¼". 5/16". 7/8".
UPRIGHT CHARACTERS
ABCDEFGHIJKLMNOPQRSTUVWXYZ.&
abcdefghijklmnopqrstuvwxyz. Plan. Elevation.
SLOPING CHARACTERS
Iron
Steel
Brick
Stone
Lead
Brass
Concrete
Timber
D° Long grain

Fig. 36

lines at right angles to the dimension lines, as shown at E (fig. 36). In dimensioning a drawing, detail dimensions should be nearest to the figure drawn, longer dimensions connecting important points should be outside these, and overall dimensions placed outside all.

Lettering and Figuring.—For lettering drawings, or, as it is termed, for "writing up", a simple legible style, free from ornate flourishes, should be adopted. The form of letters suggested in fig. 36 will be found suitable and easy to remember. No drawing or diagram should be considered complete which does not bear a descriptive title, full information as to the scale employed, and all necessary dimensions. A brief statement of the problem to which the drawing is an answer should always be appended, so that the drawing may be self-explanatory when referred to at any time, and the appearance of the work will be improved if the exercises are all enclosed in a heavy border line about $\frac{3}{4}$ in. from the edge of the paper.

Lining in.—In general, drawings should first be set out with fine lines, and these should be gone over and lined in to a uniform thickness before the dimensions are added, superfluous portions of construction lines being cleaned off. Whenever in the course of the drawing, however, lines occur which are complete and definite as to length, these should be inserted at once at their finished thickness. Where curved lines and circular arcs have to be joined to straight lines, they should be lined in first and the straight lines drawn to meet them, as it is easier to obtain a good junction between them when this order is followed.

At F and G (fig. 36) examples are given typical of errors to be avoided, and it must be borne in mind that in general it is mainly in such matters of finish, which may appear trivial to the beginner, that a good drawing shows its superiority over a moderate one.

Section Lining.—When, as frequently happens, in order to make clear hidden or internal details, parts of the object drawn are shown in section, it is usual to indicate the parts cut through in a distinctive manner. On a pencil drawing this is best done by "cross hatching" the parts in section. If a number of different materials are cut through on one section, the names of the materials should be definitely written in, using the abbreviations recommended in B.S.S. 308.

Where, as often occurs in practice, however, it is more convenient to differentiate by distinctive forms of hatching, those indicated in fig. 36 can be used; but these are not standardized, and where used should be accompanied by a key diagram explaining their meaning.

Hatching must be done carefully—generally with the 45° set square. The appearance of the drawing will be spoiled if the lines are uneven in spacing and thickness.

CHAPTER IV

SIMPLE PROBLEMS ON LINES AND ANGLES

Rigid geometrical constructions are purposely omitted wherever a practical method of sufficient accuracy can be more expeditiously used.

To Bisect a Line (fig. 37).—Open the dividers as nearly as may be judged to half the length of AB, and mark this off from each end, making *slight* indentations at C and D. Accordingly as the guess made is too great or too small, there will be an overlap or a gap CD. Bisect this small distance "by eye" at the point E, which will be, within very small limits of error, the centre of AB.

This method should be practised as being speedy, and obviating the presence of construction lines, which tend to obscure and overload a drawing. With reasonable care it is susceptible of a high degree of accuracy, and provides good eye training.

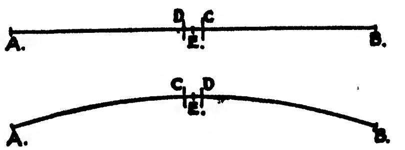


Fig. 37

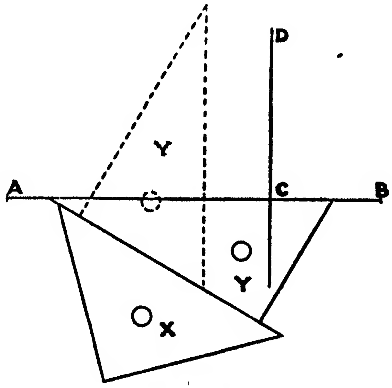


Fig. 38

The foregoing construction may also be used to bisect a circular arc.

To Draw a Line Perpendicular to Another through a Given Point in the Line.—Place the set squares X and Y together in position against AB, as shown in fig. 38. Holding X in place with the left hand, turn Y into position, as indicated by dotted lines, i.e. so that the other of the two edges containing the right angle now rests on X. Into whatever position along the edge of X the set square Y is now moved the long edge will be perpendicular to AB. Slide it into position, therefore, so that the long edge passes through the given point C, and draw the required perpendicular.

To Draw a Line Perpendicular to Another Line through a Given Point off the Line.

With the given point C as centre, describe an arc cutting AB in D and E.

With D and E as centres, describe two equal arcs intersecting at F.

CF will be the required perpendicular (fig. 39).

Practical Method.—Using the set squares as described in the previous example, slide the set square Y into position so that it passes through the given point C, and draw the perpendicular.

To Draw a Line Parallel to Another at a Given Distance away (fig. 40).—Using the set squares in the manner explained in the preceding

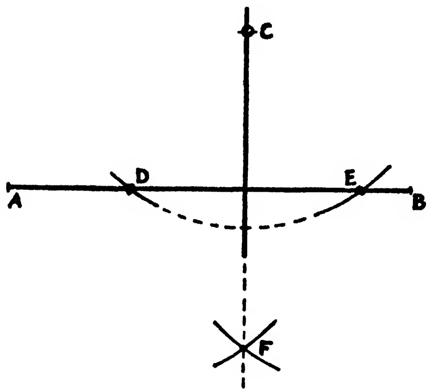


Fig. 39

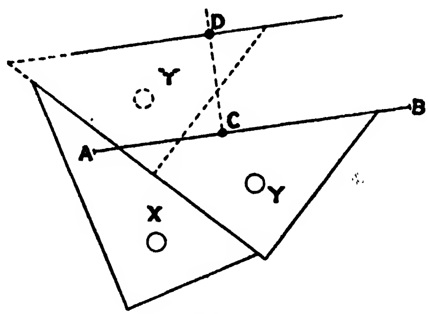


Fig. 40

examples, erect at any point C in AB a perpendicular, and on it mark out from C the length CD equal to the given distance. Placing the set squares in position, as shown in fig. 40, and holding X in place with the left hand, slide Y into position until the ruling edge passes through D, and draw the required parallel line.

This same practical method should also be employed when it is required **to draw a line parallel to a given line through a given point off the line.**

To Divide a Straight Line into a Given Number of Equal Parts.—From one end B (fig. 41) draw a faint line BC at right angles to AB. Placing the zero mark of a scale at A, and the number corresponding with the number of divisions required on BC as shown, prick or mark the paper lightly, close to the scale edge, at each of the scale divisions, and remove the scale. Through each of these points place the edge of the set square in a position parallel to BC, as shown by the dotted lines, which need not be drawn, and mark its intersection with AB.

The figure shows the method used to divide AB into 7 parts.

Facility should also be acquired in dividing lines, either straight or curved, into any number of parts by trial with the dividers (fig. 42). Estimate

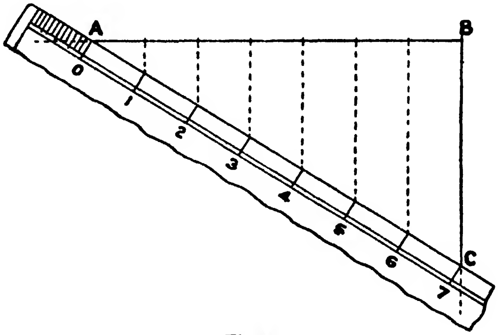


Fig. 41

" by eye " a length equal to the required fraction of the whole line AB or CD, and " step it out " along the line the required number of times without marking the paper. The first attempt will doubtless either overstep or fall short of the end of the line, as at E or F. The dividers must then be

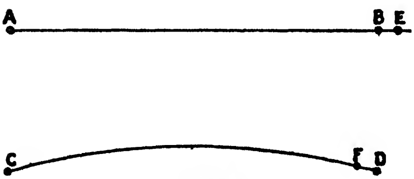


Fig. 42

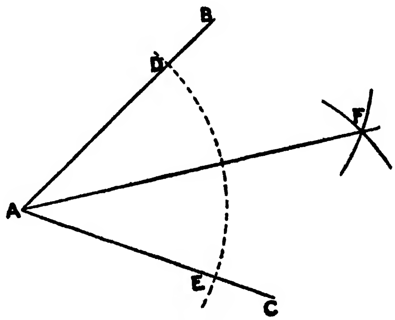


Fig. 43

opened or closed by a small amount equal to the required fraction of the remnant BE or FD, and another trial made; and so on till the correct division is found, when the points of section can then be marked off. This method will frequently be required in practice, more particularly when curved lines are to be divided.

To Bisect a Given Angle (fig. 43).—With A as centre and any radius, draw the arc DE. With D and E as centres, draw arcs of equal radii intersecting at F. AF will bisect the angle BAC.

To Divide an Angle into any Number of Equal Parts (fig. 44).—With A as centre, draw the arc DE. Divide this arc by trial into the required number of equal parts, and join the points of division to A. These radiating lines will divide the angle BAC into the required number of equal parts.

To Describe an Angle Equal to a Given Angle (fig. 45); for example, to set out from E in the line DE an angle equal to ABC.—With B as centre, describe the arc GH, and from E as centre, with the same radius, describe an arc cutting DE in J. Make JK equal to GH, using the compasses or dividers, and join EK. The angle DEK will be equal to ABC.

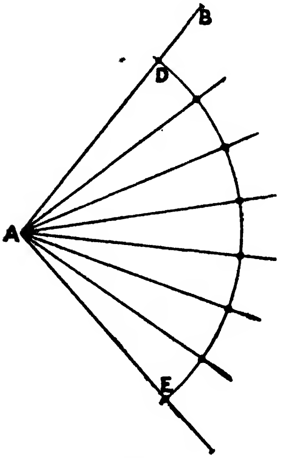


Fig. 44

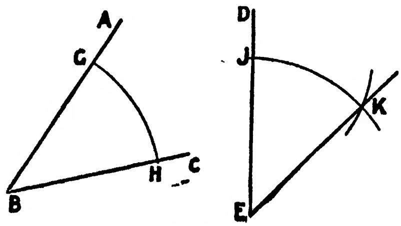


Fig. 45

EXERCISES

1. Rule a number of equidistant parallel lines, say $\frac{1}{4}$ in. apart, on tracing paper (fig. 46). Use this to divide straight lines 3 in., $2\frac{1}{2}$ in., $2\frac{3}{4}$ in. long into 7, 8, and 9 equal parts respectively. In the figure it is shown placed in position over a line AB so as to divide it into 5 equal parts, the points of crossing being pricked through.

2. Draw a line $3\frac{1}{4}$ in. long, and divide it into 3 portions in the ratio of 3, 4, and 5. (Divide into 12 parts, i.e. $3 + 4 + 5$, and take 3, 4, and 5 of these divisions respectively.)

3. Set out an angle of 93°, and divide it into 3 portions in the ratio 4 : 5 : 6. (Divide into 15 parts, and take 4, 5, and 6 of these.) Measure and write down the value of each of the 3 parts, and add your results.

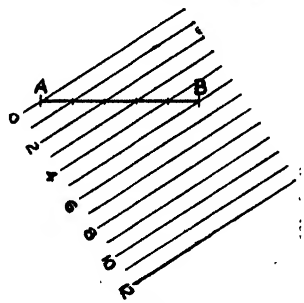


Fig. 46

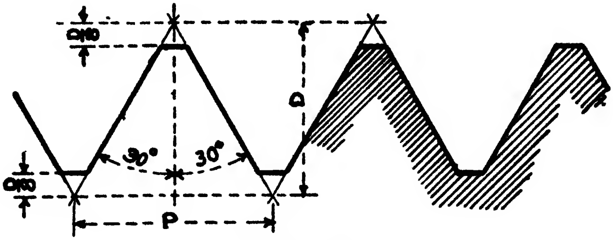


Fig. 47.

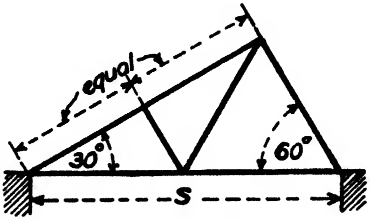


Fig. 48

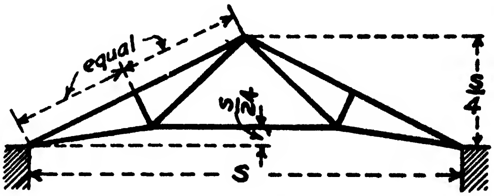


Fig. 49.

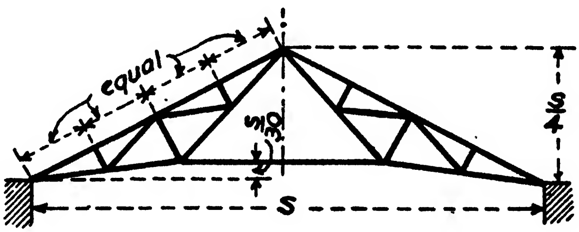


Fig. 50.

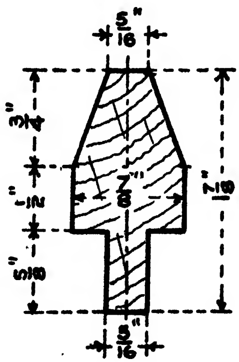


Fig. 51

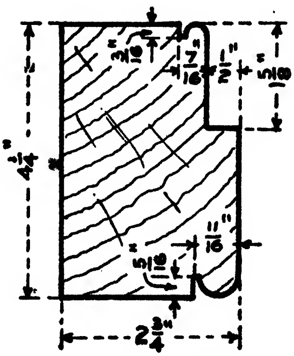


Fig. 52

4. Draw a circle 3 in. diameter, and divide the circumference into 11 equal parts by trial. Draw the radii, and with the protractor measure the angle between two that are consecutive. Check your accuracy by dividing 360° by 11.

5. Fig. 47 represents the form of the Sellers V Thread for screws, being the standard largely in use in America. Set this out full size when the pitch P (i.e. the distance apart of the threads) is $1\frac{1}{2}$ in. Measure the width of flat at the root and tip of thread, and the depth of thread from root to tip.

6. Fig. 48 gives a line diagram of a roof truss suitable for a small workshop. Set this out to a scale of $\frac{1}{2}$ in. to 1 ft. when the span S is 18 ft. Letter all the joints and tabulate the lengths of all the members.

7. Fig. 49 gives a line diagram of a roof truss. Draw the truss to a scale of 1 in. to 1 ft. when the span S is 36 ft. Tabulate the lengths of all the members, and measure the angle which the rafter makes with the horizontal.

8. Set out the line diagram of the "French" truss shown in fig. 50 to a scale of $\frac{1}{4}$ in. to 1 ft. when the span S is 60 ft. Tabulate the lengths of all the members.

9. Draw full size the section of glazing sash bar—for a wooden sash—shown in fig. 51, adding all the dimensions.

10. Fig. 52 gives a section of a solid rebated and beaded timber, as used for a door frame, the size being known commercially as $4\frac{1}{2}$ in. × 3 in. (actually $4\frac{1}{4}$ in. × $2\frac{3}{4}$ in.). Draw this to a scale of half full size, and dimension it completely.

CHAPTER V

THE CIRCLE

Simple Problems connected with the Circle

Figs. 53 and 54 illustrate the following definitions:

Circle.—A plane figure bounded by a curved line which is at all points at a constant distance from an internal point called the **centre.**

Circumference.—The curved boundary of a circle.

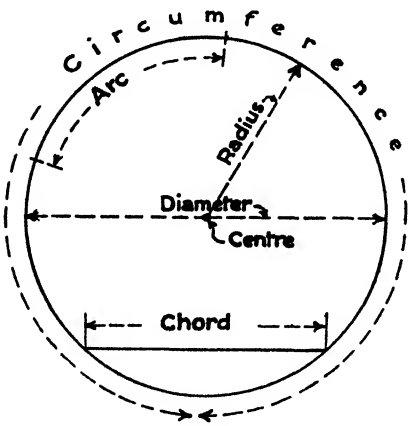


Fig. 53

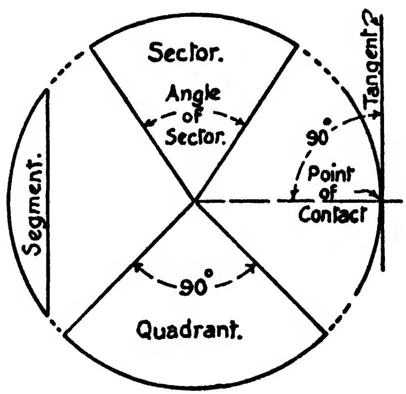


Fig. 54

Radius.—A straight line drawn from the centre to the circumference.

Diameter.—A straight line through the centre terminated at each end by the circumference.

Chord.—A straight line joining any two points on the circumference. A diameter is the longest chord which can be drawn in a circle.

Arc.—Any portion of the circumference of a circle.

Sector.—A portion of a circle bounded by two radii and the included arc.

Angle of a Sector.—The angle between the two bounding radii.

Quadrant.—A sector in which the angle between the two bounding radii is a right angle; its area is evidently $\frac{1}{4}$ that of the circle of which it is a part.

Semicircle.—One of the two equal portions into which a circle is divided by a diameter; or, a sector having an angle of 180°.

Segment.—A portion of a circle bounded by an arc and a chord.

Angle in a Segment.—The angle between two lines joining any point on the arc to the ends of the chord of the segment.

Angle at the Centre.—The angle between two radii drawn from the extremities of any arc or chord.

Tangent.—A straight line which touches a circle at a single point in its length, and is at right angles to the radius through that point.

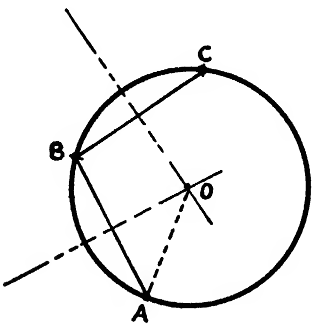


Fig. 55

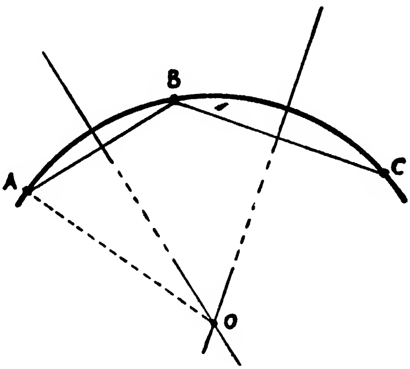


Fig. 56

Point of Contact.—The point at which the tangent touches the circle. Also the point at which two circles or curves touch each other.

The following selected fundamental constructions are of constant occurrence in practical drawing and should be worked through, several examples of each case being taken with the given conditions varied each time:

To Find the Centre of a Given Circle or Circular Arc (figs. 55 and 56).—Choose any three points A, B, and C on the circumference. Bisect AB and BC by lines at right angles. These will intersect at O, the required centre, a line from which, such as OA, will give the length of radius.

To Draw a Circular Arc through Three Given Points (fig. 56).—The location of the centre follows directly from the foregoing, A, B, and C being the three given points and O the required centre.

To Describe a Circle about a Given Triangle.—This is obviously

a particular case of the foregoing. The circle through the three corners of a triangle is called the **circumscribed circle.**

To Inscribe a Circle in a Given Triangle (fig. 57).—The three

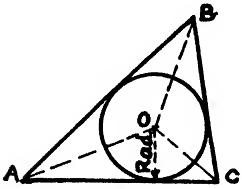


Fig. 57

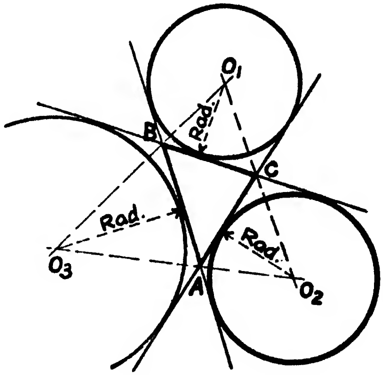


Fig. 58

bisectors of the angles meet at the required centre O. In practice only two need be drawn. A perpendicular from O to any side gives the radius.

To Draw the Escribed Circles to a Given Triangle (fig. 58).—These circles touch any side and the other two sides produced. The bisectors of the external angles meet at the required centres O_1, O_2, and O_3.

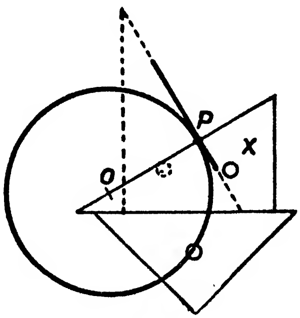


Fig. 59

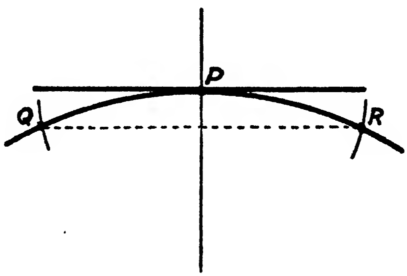


Fig. 60

To Draw a Tangent to a Circle through a Point P in the Circumference (fig. 59).—Place the set squares as shown to pass through O (the centre) and P (the given point)—OP need not be drawn unless required. Reverse the set square X, as shown by dotted lines, so as to draw through P a line at right angles to OP. This will be the required tangent and P its point of contact.

To Draw a Tangent to a Circular Arc at a Given Point P without using the Centre (fig. 60).—With P as centre and convenient radius describe arcs to cut the given arc in Q and R. Through P draw a line parallel to QR. This will be the required tangent. A line through P perpendicular to QR will be radial.

To Draw a Tangent to a Circle from an External Point P (fig. 61).—Join O to P, and on this as diameter describe a circle cutting the given circle in R and S. These will be the points of contact, and PR and PS the required tangents.

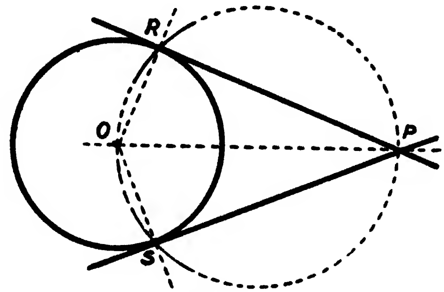


Fig. 61

To Draw the Common Tangents to Two Given Circles (fig. 62).—In general four can be drawn, two internal and two external.

The circles have their centres at O_1 and O_2 and their radii are a and b. From O_1 describe a circle of radius $a - b$. By the construction in the preceding case draw tangents to this from the point O_2 and the radii through the points of contact, produced to P_1 and P_2. Lines parallel to these tangents through P_1 and P_2 will be the two *external tangents.*

From O_1 describe a circle of radius $a + b$. Draw tangents to this from the point O_2 and the radii through the points of contact cutting circle A in R_1 and R_2. Lines parallel to these tangents through R_1 and R_2 will be the two *internal tangents.*

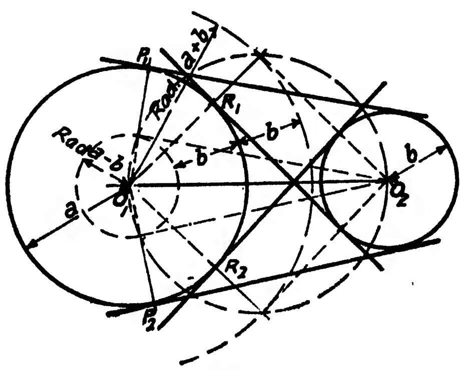


Fig. 62

To Draw a Circle of Known Radius to touch Two Given Straight Lines (fig. 63).—Let AB and AC be the given lines. Draw lines parallel to these at a distance equal to the given radius, intersecting at O. Locate the points of contact D and E by drawing perpendiculars to AB and AC through O. With OD or OE as radius describe the required circle. In all cases AO bisects the angle BAC.

To Draw a Circle of Known Radius to touch a Given Line and a Given Circle (fig. 64).—Draw a parallel to the given line AB at a distance equal to the known radius of the required circle. From O with

radius equal to the sum of the radii of the given and required circles describe an arc to cut this line at P. Join O and P, cutting the given circle at D. Draw PC perpendicular to AB. P will be the centre, and PC or PD the radius of the required circle.

To Draw a Circle of Known Radius to touch Two Given Circles

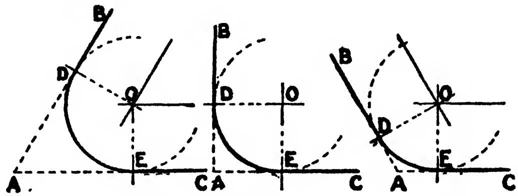


Fig. 63

or Arcs (fig. 65).—Call the radii of the two given circles or arcs *a* and *b* respectively, and the known radius of the required circle *r*.

With centre A and radius $a + r$ describe an arc. With centre B and radius $b + r$ describe a second arc cutting the first at P. Join A and B to P, cutting the given circle in C and D. P will be the centre, and PC or PD the radius of the required circle.

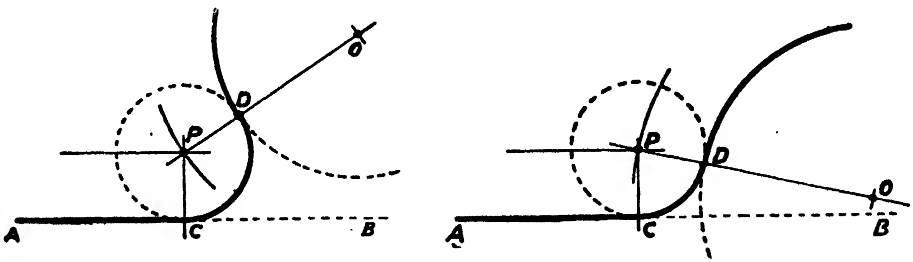


Fig. 64

Note.—In practice the drawing of connecting arcs such as those in the three foregoing examples is of frequent occurrence, and unless the radii dealt with are large they are generally put in without construction, the position of the centre being located by trial. This requires care and experience, and should not be attempted until the constructions are known and mastered. The centres can also be readily found if a circle of the required radius is drawn on tracing paper, adjusted to position over the drawing, and the centre pricked through, this method being expeditious and thoroughly practical.

To Draw a Circle to touch Two Given Lines, the Point of Contact on One being Given (fig. 66).—Through P, the given point of contact, draw a perpendicular to AB. Bisect the angle between AB and

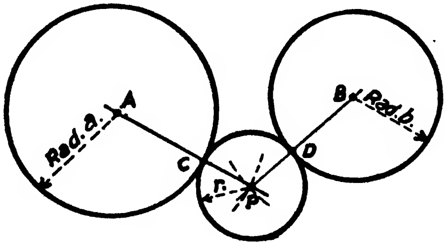


Fig. 65

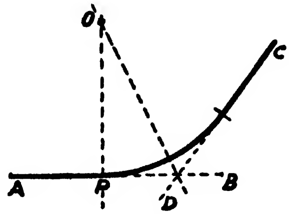


Fig. 66

CD. The two lines so drawn will intersect at O, the centre of the required circle, of which OP is the radius.

When Two Circles are in Contact the Line joining their Centres Passes Through the Point of Contact.—If, at the point of contact, a line is drawn perpendicular to the line joining their centres, this will be a

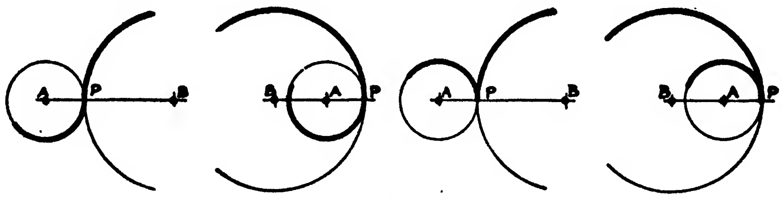


Fig. 67

tangent to both of the circles, or they are said to have a common tangent. Two arcs are said to meet tangentially when they have a common tangent at their point of contact. Thus in fig. 67 the arcs shown by heavy lines meet tangentially at the point P, which point lies on the line joining the centres of the arcs.

EXERCISES

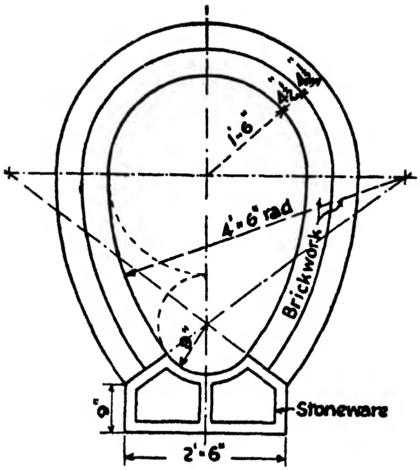


Fig. 68

1. Fig. 68 shows a cross-section of an egg-shaped brick sewer, with a stoneware invert. Set out a section to a scale of 1 in. to 1 ft.

2. A section of a sash bar of the form known as "ovolo and fillet" is given in fig. 69. Draw full size, showing how you arrive at the centres for the ovolo curves.

3. Fig. 70 gives a cross-section of a $1\frac{1}{4}$ in. panel moulding for a moulded door. Draw twice full size.

4. Draw a full-size section of the "bolection" moulding suitable for an external door shown in fig. 71.

5. Draw a full-size section of the moulded hardwood handrail shown in fig. 72, finding the centres of the circular arcs by construction.

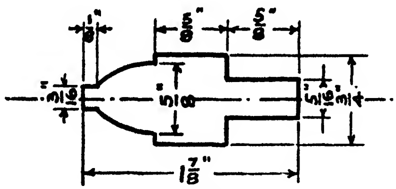


Fig. 69

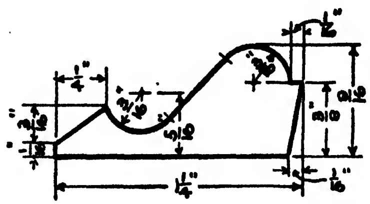


Fig. 70

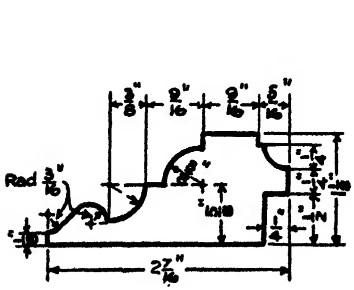


Fig. 71

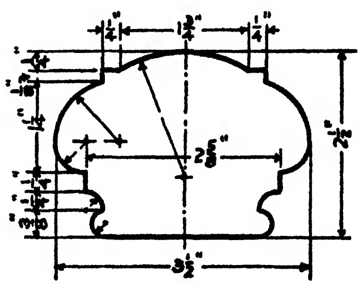


Fig. 72

6. Fig. 73 gives the outline of a form of eye suitable for a mild-steel tie-bar. Dimensions are given in terms of the width of the tie-bar, i.e. the overall width is 2·5 times the width of bar, and so on. Draw $\frac{1}{4}$ full size and dimension eyes suitable for tie-bars 4 in. and 6 in. wide respectively.

7. Fig. 74 also gives a suitable form of eye for the end of a tie-bar, dimensions being given in terms of the width of the bar. Draw $\frac{1}{4}$ full size and dimension eyes suitable for tie-bars $3\frac{1}{2}$ in. and 5 in. wide respectively.

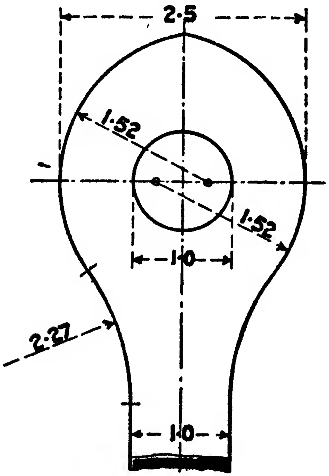


Fig. 73

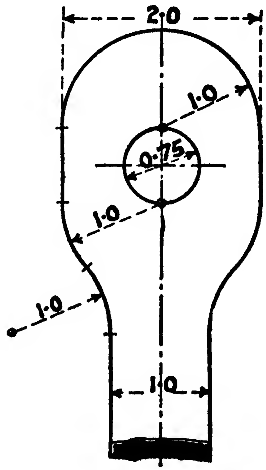


Fig. 74

8. Fig. 75 shows the British Standard section for rolled-steel equal sided angles. Draw sections of the following angles to the given dimensions:

Section	A	B	T	R_1	R_2
$3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$ angle	$3\frac{1}{2}''$	$3\frac{1}{2}''$	$\frac{1}{2}''$	·325″	·225″
$6'' \times 6'' \times \frac{5}{8}''$,,	6″	6″	$\frac{5}{8}''$	·475″	·325″

9. Fig. 76 shows the British Standard section for rolled-steel unequal sided angles. Draw sections of the following angles to the given dimensions:

Section	A	B	T	R_1	R_2
$4'' \times 3'' \times \frac{7}{16}''$ angle	4″	3″	$\frac{7}{16}''$	·325″	·225″
$6'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$,,	6″	$3\frac{1}{2}''$	$\frac{1}{2}''$	·400″	·275″

10. Fig. 77 shows the British Standard section for rolled T bars. Draw the following sections full size:

Section	A	B	T	R_1	R_2
4″ × 3″ × ⅜″ T 6″ × 3″ × ½″ ,,	4″ 6″	3″ 3″	⅜″ ½″	·325″ ·40″	·225″ ·275″

11. Fig. 78 shows the British Standard section for rolled-steel channels. Draw the following sections half full size:

Section	A	B	T_1	T_2	R_1	R_2
8″ × 3½″ × 22·7 lb. per foot 10″ × 4″ × 30·2 ,, ,,	8″ 10″	3½″ 4″	·425″ ·475″	·525″ ·575″	·525″ ·575″	·375″ ·40″

12. Fig. 79 shows the British Standard section for rolled-steel joists. Draw the following sections half full size:

Section	A	B	T_1	T_2	R_1	R_2
6″ × 5″ × 25 lb. per foot 12″ × 6″ × 54 ,, ,,	6″ 12″	5″ 6″	·41″ ·50″	·52″ ·88″	·51″ ·60″	·255″ ·30″

13. Fig. 80 shows the form of section for rolled-steel Broad Flange Beams. Draw the following sections half full size:

Section (Nominal Dimensions)	A Actual	B Actual	T_1	T_2	R
7″ × 7″ × 31·5 lb. per foot 16″ × 12″ × 107 ,, ,,	7⅛″ 15¾″	7⅛″ 11$\frac{13}{16}$″	·33″ ·61″	·66″ 1·22″	·33″ ·61″

14. Fig. 81 shows the British Standard section for rolled Z beams. Draw the following sections half full size:

Section	A	B	C	T_1	T_2	R_1	R_2
6″ × 3½″ Z 8″ × 3½″ ,,	6″ 8″	3½″ 3½″	3½″ 3½″	·375″ ·425″	·475″ ·525″	·425″ ·45″	·30″ ·325″

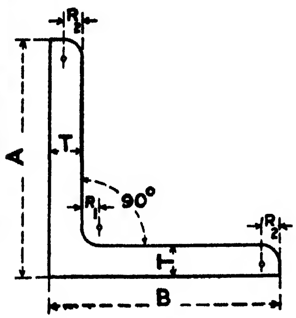


Fig. 75

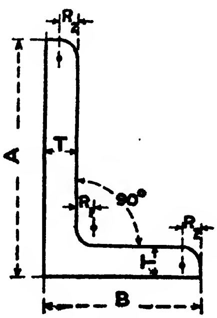


Fig. 76

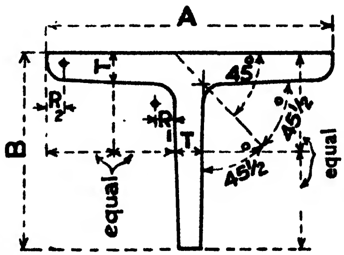


Fig. 77

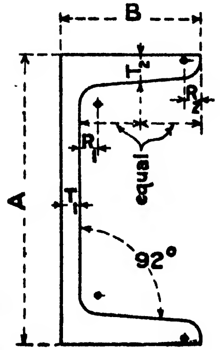


Fig. 78

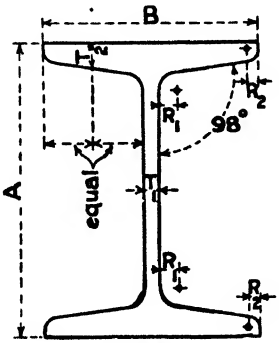


Fig. 79

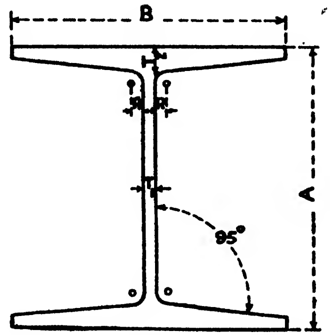


Fig. 80

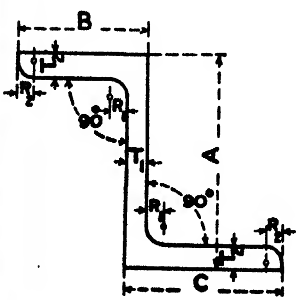


Fig. 81

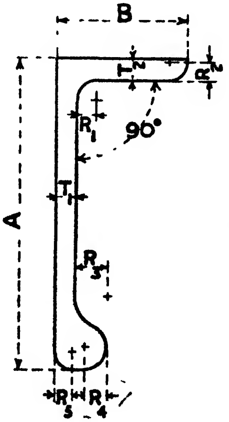


Fig. 82

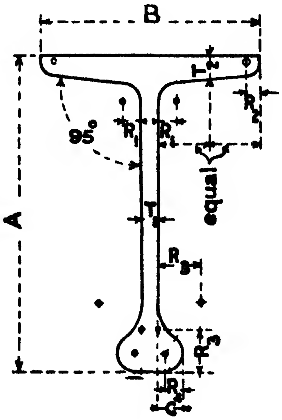


Fig. 83

15. Fig. 82 shows a British Standard section of rolled-steel bulb angle. Draw the following sections full size:

Section	A	B	T_1	T_2	R_1	R_2	R_3	R_4	R_5
6″ × 3″, Bulb L	6″	3″	$\frac{3}{8}$″	$\frac{3}{8}$″	·40″	·275″	·675″	·40″	·325″
7″ × 3½″, ,,	7″	3½″	$\frac{7}{16}$″	$\frac{7}{16}$″	·45″	·30″	·75″	·45″	·375″

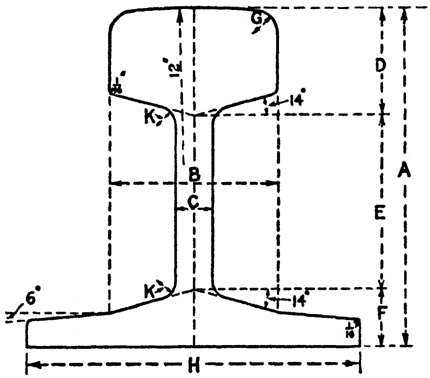


Fig. 84

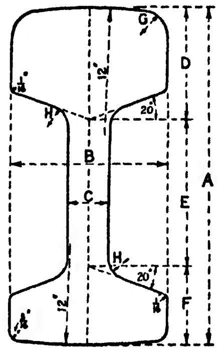


Fig. 85

16. Fig. 83 shows a British Standard section of rolled-steel bulb T. Draw the following sections full size:

Section	A	B	C	T_1	T_2	R_1	R_2	R_3	R_4
7″ × 5″, Bulb T	7″	5″	·45″	·425″	·425″	·60″	·20″	·80″	·30″
8″ × 5½″, ,,	8″	5½″	·50″	·45″	·45″	·675″	·225″	·90″	·325″

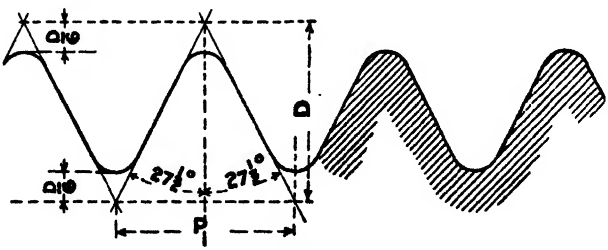


Fig. 86

17. Fig. 84 shows the British Standard section for flat-bottomed rails. Draw the following sections full size:

Section	A	B	C	D	E	F	G	H	K
60 lb. per yard	$4\frac{5}{16}''$	$2\frac{1}{4}''$	$\frac{29}{64}''$	$1\frac{13}{32}''$	$2\frac{3}{16}''$	$\frac{23}{32}''$	$\frac{11}{32}''$	$4\frac{5}{16}''$	$\frac{7}{32}''$
80 ,, ,,	$5''$	$2\frac{1}{2}''$	$\frac{35}{64}''$	$1\frac{19}{32}''$	$2\frac{9}{16}''$	$\frac{27}{32}''$	$\frac{3}{8}''$	$5''$	$\frac{1}{4}''$

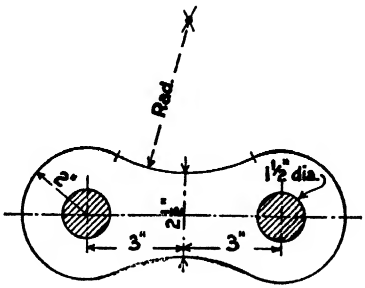


Fig. 87

18. Fig. 85 shows the British Standard section for bull-head rails. Draw the following sections full size:

Section	A	B	C	D	E	F	G	H
85 lb. per yard	$5\frac{15}{32}''$	$2\frac{11}{16}''$	$\frac{11}{16}''$	$1\frac{3}{4}''$	$2\frac{15}{32}''$	$1\frac{1}{4}''$	$\frac{1}{2}''$	$\frac{1}{4}''$
100 ,, ,,	$5\frac{29}{32}''$	$2\frac{3}{4}''$	$\frac{3}{4}''$	$2\frac{3}{32}''$	$2\frac{15}{32}''$	$1\frac{11}{32}''$	$\frac{1}{2}''$	$\frac{1}{4}''$

19. The form of Whitworth standard V thread is illustrated in fig. 86. Draw 8 times full size the section of screw thread when the "pitch" P is $\frac{1}{4}$ in., i.e. the distance P on your drawing will be represented by 2 in.

20. Fig. 87 represents a link of a pitch chain. Draw $\frac{1}{2}$ full size a length of three complete links assembled in position.

CHAPTER VI

THE TRIANGLE

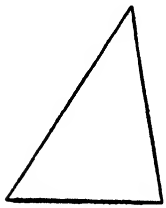
A triangle is a plane figure bounded by three straight lines.

Triangles are known by distinctive names, according to their shape. Thus:

An **acute-angled triangle** (fig. 88) has three acute angles, that is, each of them is less than 90°.

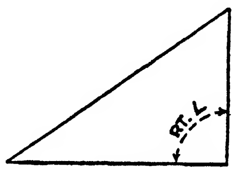
A **right-angled triangle** (fig. 89) has one angle 90°.

An **obtuse-angled triangle** (fig. 90) has one angle greater than 90°.



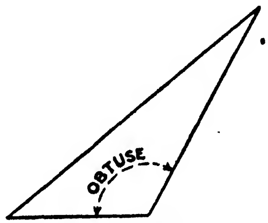
ACUTE ANGLED

Fig. 88



RIGHT ANGLED

Fig. 89



OBTUSE ANGLED

Fig. 90

An **isosceles triangle** has two of its sides equal. It may be acute (fig. 91), equilateral when it has three equal sides and three equal angles (fig. 92), right-angled (fig. 93), or obtuse-angled (fig. 94).

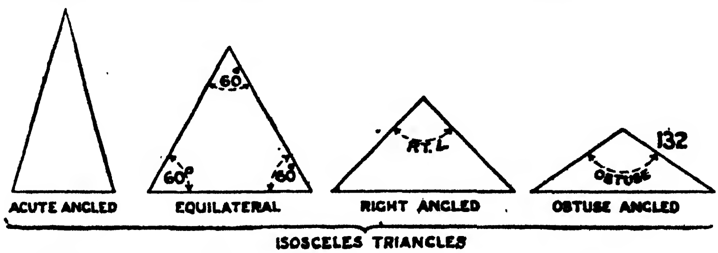


Fig. 91 Fig. 92 Fig. 93 Fig. 94

In any triangle there are six items which may be specified, three sides and three angles. For brevity, these are distinguished as shown in fig. 95, the angles being denoted by capital letters A, B, and C, and the sides by

small letters, it being noted that the side *a* is opposite the angle A, and so on. If we have any three of the six items we have sufficient information to enable a triangle to be constructed, except in the case where the three angles are stated.

Construction of Triangles from given Data

Given the Three Sides *a*, *b*, and *c* (fig. 96).—Set out one side, say *a*. From one end with radius *b* describe an arc, and from the other end with radius *c* describe a second arc. This will cut the first in the point A, the third corner of the triangle.

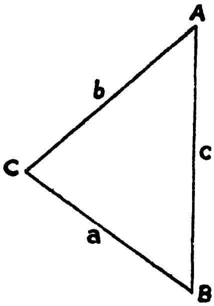


Fig. 95.—The above shows the usual method of lettering a triangle for reference, the sides being referred to as *a*, *b*, and *c*, and the angles as A, B, and C.

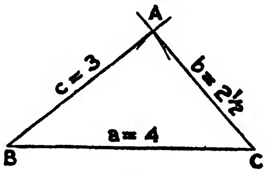


Fig. 96.—Construction of triangle given 3 sides; for example, $a = 4$, $b = 2\frac{1}{2}$, $c = 3$.

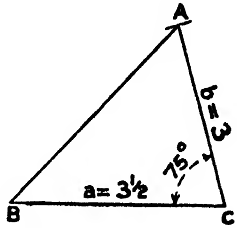


Fig. 97.—Construction of triangle given 2 sides and included angle; for example, $a = 3\frac{1}{2}$, $b = 3$, $C = 75°$.

Given Two Sides, say *a* and *b*, and the included Angle C (fig. 97).—Set out one side, say *a*. From one end set out a line making an angle C with this, and on it measure the length *b*, and draw in the third side.

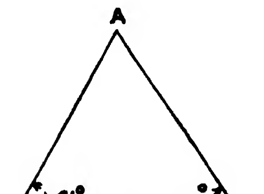


Fig. 98.—Construction of triangle given 1 side and 2 adjacent angles; for example, $a = 4$, $B = 61°$, $C = 55°$.

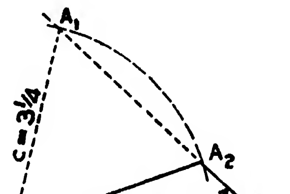


Fig. 99.—Construction of triangle given 2 sides and 1 opposite angle; for example, $a = 4\frac{1}{4}$, $c = 3\frac{1}{4}$, $C = 42°$—the ambiguous case.

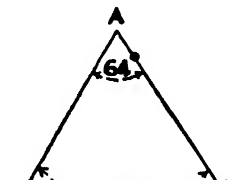


Fig. 100.—Construction of triangle given 1 side and the opposite and one adjacent angle; for example, $a = 3\frac{3}{4}$, $A = 64°$, $B = 60°$.

Given One Side *a* and the Two Adjacent Angles B and C (fig. 98).—Set out the side *a*. From one end set out a line making an angle B with the side *a*, and from the other a line making an angle C with the side *a*. These lines will intersect in the third corner A.

Given Two Sides *a* and *c* and One Opposite Angle, say C (fig. 99).—Set out one side *a*, lettering the ends B and C, and from the end C set out a line making an angle C with it. From the other end B as centre, describe an arc of radius *c* to cut this line. In general it will be found that this arc cuts the line in two points A_1 and A_2, showing that there are two triangles A_1BC and A_2BC which satisfy the conditions. For this reason this is known as the **ambiguous case.**

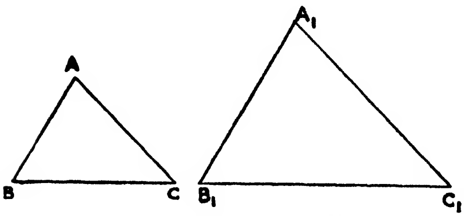


Fig. 101.—Similar Triangles

Given One Side *a* and Two Angles A and B (fig. 100).—Since the three angles of any triangle are equal to 180°, the third angle C may be found by subtracting A and B from 180°, and the triangle constructed by the method already described for the case where *a*, B, and C are given.

When the **three angles** of a triangle are given, it will be seen that this imposes no limit on the size of the triangle. For example, the two triangles ABC, $A_1B_1C_1$ in fig. 101 have their angles equal, but are not of the same size, and do not enclose the same area. The two triangles, while not identical, resemble each other, and are said to be **similar.**

EXERCISES

Construct triangles from the following data. Measure and write down the values of the remaining sides and angles in each case:

1. $a = 3$ in., $b = 4$ in., $c = 2$ in.
2. $a = 6$ ft. 7 in., $b = 3$ ft. 4 in., $c = 5$ ft. 9 in. Scale 1 in. = 1 ft.
3. $a = 2\frac{3}{4}$ in., $b = 4\frac{1}{4}$ in., C = 60°.
4. $c = 2$ ft. 3 in., $a = 2$ ft. 6 in., B = 95°. Scale $\frac{1}{8}$ full size.
5. $a = 3\frac{1}{4}$ in., B = 75°, C = 64°.
6. $c = 5$ ft. 9 in., A = 45°, B = 82°. Scale $\frac{3}{4}$ in. = 1 ft.
7. $a = 4$ in., $c = 3$ in., C = 45°.
8. $b = 6$ ft. 2 in., $a = 4$ ft. 8 in., A = 47°. Scale $\frac{1}{16}$ full size.
9. $a = 3\frac{1}{2}$ in., A = 70°, B = 51°.
10. $c = 9$ ft. 8 in., C = 65°, A = 55°. Scale $\frac{1}{2}$ in. = 1 ft.
11. Set out a triangle, sides *a*, *b*, and *c* being $2\frac{1}{2}$ in., $3\frac{3}{4}$ in., and $3\frac{1}{8}$ in. respectively. Draw a similar triangle in which the side *a* is 3 in. long. Measure and write down the lengths of the sides *b* and *c*.

12. A man, whose eye is at a height of 5 ft. 6 in. above ground level, observes the top of a distant flagstaff, and his line of sight makes an angle of 30° with the horizontal. He walks towards the flagstaff, a distance of 25 ft., and then finds his line of sight to make 45° with the horizontal. By setting out to a suitable scale, find the height of the flagstaff.

13. A man on a tower 40 ft. high observes two objects in line with the foot of the tower on the level ground below. If his line of sight makes an angle with the horizontal of 60° for the nearer object and 34° for the farther one, how far are the objects apart?

Right-angled Triangles.—In the right-angled triangle ABC (fig. 102) AB is the " base ", BC the " perpendicular ", and AC the " hypotenuse ".

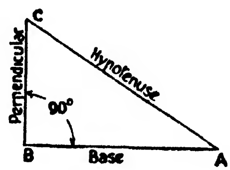


Fig. 102

Note that in a right-angled triangle if one angle is known the other is found by subtracting the known angle from 90°, the two angles being said to be **complementary.**

Construction of Right-angled Triangles from given Data

Given the Base and Perpendicular.—On two lines at right angles set out the given lengths, and join the extremities to give the hypotenuse.

Given the Hypotenuse and One Side.—Set out two lines at right angles, measure the given side on one, and from its end as centre, with radius equal to the hypotenuse, describe an arc to cut the other in the required third corner of the triangle.

Given the Hypotenuse and an Angle.—Set out two lines containing the given angle; on one measure the hypotenuse, and from its extremity draw a perpendicular to the other to meet it in the required third corner of the triangle.

In any Right-angled Triangle the Square described on the Hypotenuse is Equal to the Sum of the Squares described on the Base and Perpendicular.—This may be verified graphically (fig. 103). Draw the squares on the three sides. Through C draw DE parallel to AB and through D draw DF perpendicular. Cut the two squares on AC and BC along the lines DE and DF into 5 pieces as shown. These 5 pieces can then be reassembled to fill exactly the square on the hypotenuse AB as shown by the dotted lines.

This property of the right-angled triangle can be further extended by stating that the area of any figure described on the hypotenuse is equal

to the combined area of two similar and similarly placed figures described on the base and perpendicular respectively.

To Reduce a Triangle to an Equal Triangle with a Given Length as Base (fig. 104).—From B mark out on the base line BC the given length of base BD. Join D to A, and through C draw a parallel to meet AB in E. The triangle EBD will be equal in area to ABC.

To Reduce a Triangle to an Equal Triangle of Given Altitude (fig. 105).—Set out a line parallel to the base BC at a distance away equal to the given altitude, to cut AB (produced if necessary) at E. Join EC, and draw the parallel AD. The triangle EBD will be equal in area to ABC.

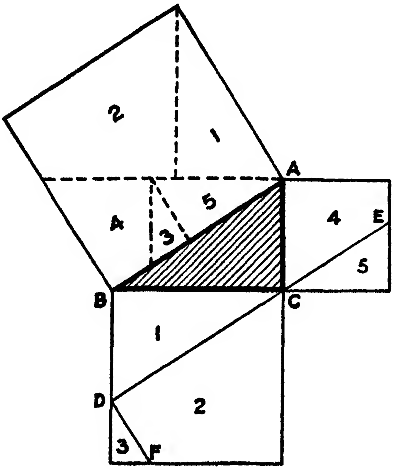


Fig. 103

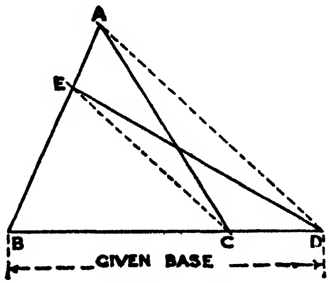


Fig. 104

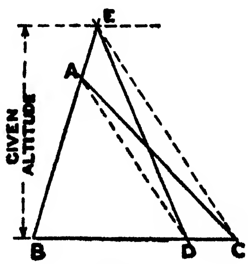


Fig. 105

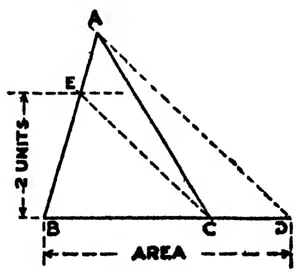


Fig. 106

To Determine the Area of a Triangle Graphically (fig. 106). —Reduce the triangle to an equal triangle having an altitude of 2 units by the method given above. The length of the base of this equivalent triangle when measured in the same units as are employed for the altitude will give the area. Thus if the area is required in square inches, reduce

the triangle to one having an altitude of 2 in. The length of the base of this triangle in inches will equal the number of square inches in its area. If the area is required in square centimetres, the altitude must be made 2 cm., and the base measured in centimetres, and so on.

EXERCISES

14. Set out to any suitable scales triangles having sides in the following ratios:

$$3 : 4 : 5, \quad 5 : 12 : 13, \quad 8 : 15 : 17, \quad 7 : 24 : 25, \quad 20 : 21 : 29.$$

Test the greatest angle in each case—it should be a right angle.

15. A man stands at the top of a vertical tower 50 ft. high and sights a rock on the plain below. His line of sight makes 31° with the horizontal. How far is the rock from the tower?

16. What length of ladder would be required to reach a window 21 ft. from the ground, the foot of the ladder being 9 ft. from the wall? What angle would it make with the ground?

17. What length of shadow would be cast by a stick 7 ft. 3 in. high when the sun is 32° above the horizon?

18. In each of the given $\triangle$s L and M (fig. 107), which are to be drawn to the sizes given, draw—

(*a*) The three lines bisecting the angles.

(*b*) The three lines bisecting the sides at right angles.

(*c*) The three lines joining the corners to the mid-points of the opposite sides. These lines are termed **medians**.

(*d*) The three lines drawn from the corners perpendicular to the opposite sides.

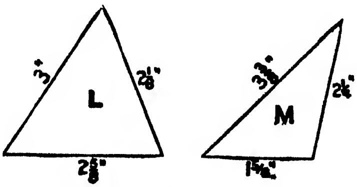


Fig. 107

Note that in each case the three lines pass through a common point.

19. A plumber has to supply a pipe to take the combined full flow from two branch pipes each 1¼ in. inside diameter, or as it is termed 1¼ in. bore. By setting out a right-angled triangle, determine the diameter of pipe necessary. If the market sizes of pipes between 1 in. and 2 in. diameter proceed by steps of ¼ in., state the nearest practicable size.

20. Give the theoretical and practical sizes of pipe necessary to take the combined flow of two pipes 1¾ in. and 2¼ in. bore respectively, the sizes of pipes obtainable proceeding by steps of ¼ in.

21. Fig. 108 represents a piece of 7-lb. lead (i.e. lead weighing 7 lb. per square foot). Determine its area graphically, and calculate its weight.

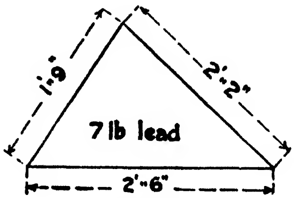


Fig. 108

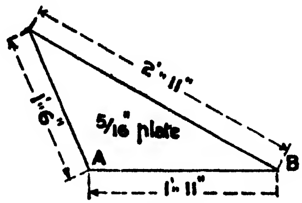


Fig. 109

22. Fig. 109 represents a mild-steel gusset plate, $\frac{5}{16}$ in. thick, weighing 12·75 lb. per square foot. Determine its area graphically, and calculate its weight.

CHAPTER VII

PLANE FIGURES

The Rectangle.—A four-sided figure in which all the angles are right angles. Opposite sides are equal in length, and if the sides are a and b, the area $= a \times b$.

To Determine the Area of a Rectangle Graphically (fig. 110).—Let ABCD be the rectangle the area of which is required.

Set up from D on AD a length ED of one unit (i.e. 1 in. if the area is required in square inches; 1 ft. if the area is required in square feet). Join EC and draw the parallel AG. The rectangle DEFG will be equal to ABCD, and the number of units in the base DG will give the area in square units.

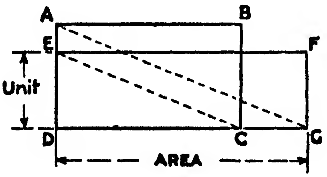


Fig. 110

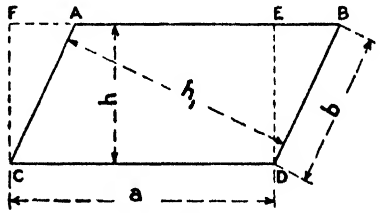


Fig. 111

The Parallelogram.—A four-sided figure in which the opposite sides are equal to and parallel with each other. Verify by actual trial:—

(*a*) That the longer diagonal joins the two opposite acute angles.

(*b*) The opposite angles are equal.

(*c*) The diagonals bisect each other.

(*d*) Each diagonal divides the figure into two equal and similar triangles.

(*e*) The two diagonals divide the figure into two pairs of equal similar triangles, all four being equal in area.

The Area of a Parallelogram is found by Multiplying a Side by its Perpendicular Distance from the Opposite Side.—Thus in fig. 111 the area of ABDC is equal to $a \times h$ or to $b \times h_1$.

The area of the figure is equal to the rectangle CDEF, and may be found graphically by applying the construction given above to the rectangle CDEF.

The Quadrilateral.—This name may be given to any four-sided figure, but is generally restricted to an irregular figure, such as ABDC (fig. 112).

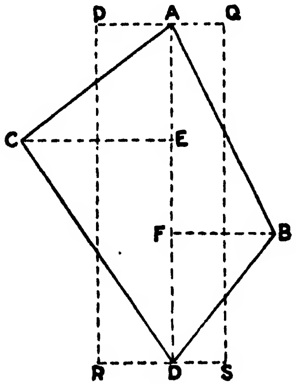


Fig. 112

If CE and BF are bisected, and the rectangle PQSR drawn, this will be equal in area to the quadrilateral, and the area may be found graphically by applying the construction given above to the rectangle PQSR.

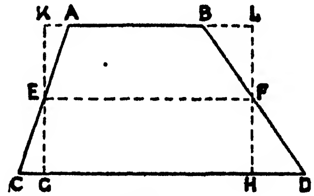


Fig. 113

When two sides of a quadrilateral are parallel, the figure is called a **trapezoid** (or **trapezium**) (ABDC, fig. 113).

If AC and BD are bisected in E and F, and the rectangle GHLK drawn, this will be equal in area to the trapezoid.

The **area is equal to** EF × GK, i.e. to **half the sum of the parallel sides multiplied by the perpendicular distance between them.**

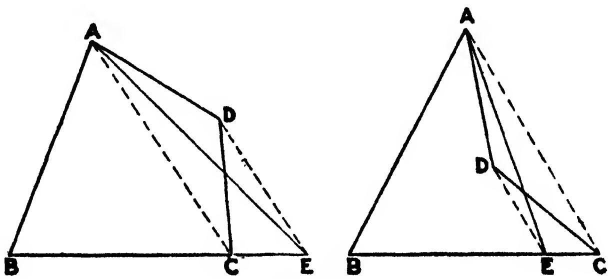


Fig. 114

To find the area graphically, apply the graphic method to find the area of the equivalent rectangle GHLK.

To Describe a Triangle Equal in Area to a Quadrilateral (fig. 114).—Two cases are illustrated, the second having a re-entrant angle at D.

Join A to C, and through D draw DE parallel thereto, to cut BC

(produced if necessary) in E. The triangle ABE will be equal to the quadrilateral ABCD.

To Describe a Square Equal in Area to a Given Rectangle (fig. 115).—Let ABCD be the rectangle. Produce AB and set out BE equal to BC. Describe a semicircle on AE as diameter. Produce CB to cut the semicircle in F. BF will be the side of the required square.

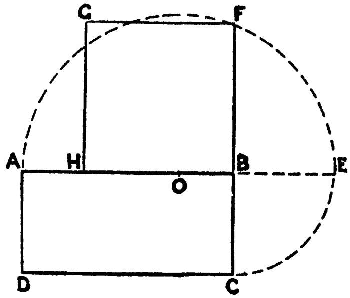


Fig. 115

To Reduce an Irregular Figure, such as ABCDEF (figs. 116 and 117), **to a Triangle of Equal Area.**—Join FD, cutting off the triangle DEF, and draw GE parallel. Then the five-sided figure ABCGF is equal to the original figure. Now join AG and draw FH parallel. Then the four-sided figure ABCH is equal to ABCGF. Lastly, join AC and draw BK parallel. The triangle AKH is equal in area to ABCH and therefore to the original figure ABCDEF.

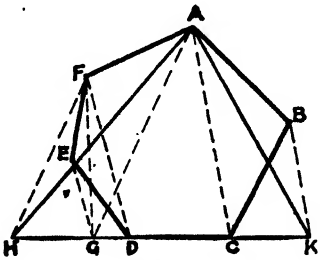


Fig. 116

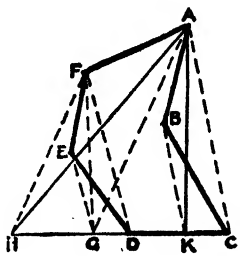


Fig. 117

When an area is bounded by irregular outlines, such as ABCDEF (fig. 118), an approximation to the area may be arrived by the **mid-ordinate rule.** Thus:—

Divide the figure as shown into any number of parallel strips of equal width. Measure the lengths 1, 2, 3, 4, &c., of these strips at the centre, as shown by dotted lines. Divide the sum of these lengths by the number of strips. The product of this and the length L will give the area approximately.

$$\text{Or; Area} = \frac{\text{sum of mid-ordinates}}{\text{number of strips}} \times \text{L}.$$

Another method giving a reasonably close approximation is known as the **trapezoidal rule.**

Divide the figure as above into any number of parallel strips of equal width by means of ordinates, as shown by full lines in fig. 118. Measure the end ordinates AD and CF, to these add twice the sum of all the other ordinates. Divide the total by twice the number of strips into which the figure is divided. The product of this and the length L will give the area.

$$\text{Or; Area} = \frac{\text{(sum of end ordinates)} + \text{(twice sum of remaining ordinates)}}{\text{twice the number of strips}} \times \text{L}.$$

With care and judgment close results may be obtained by the use of what are known in practice as **give-and-take lines.** These are illustrated at PQ and RS (fig. 118), and must be put in by estimation, so that the portions of area cut off on one side of the line are equal as nearly as may be judged to the portions of area added on the other side, the give-and-take line averaging as it were the irregularities of the outline. The area of the figure ABCDEF will then be equal to $\frac{1}{2}$(PR + QS)L. This method should be persevered with, as it is largely used in practice. In order to see both sides of the give-and-take lines at once, a black thread or a line drawn on tracing paper may be used and adjusted to position before ruling the line in.

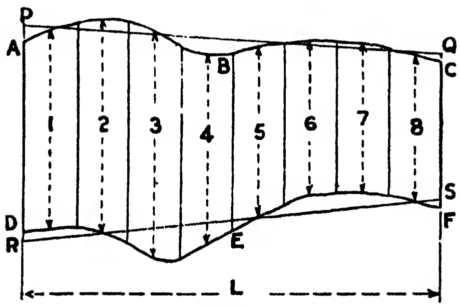


Fig. 118

Practice may be obtained by computing the area of some irregular figures by each of the three methods given and comparing the results.

EXERCISES

1. Fig. 119 shows a window opening. It is required to replace this by a square window admitting the same amount of light. Set out the size of the square window.

2. Set out to a scale of 1 in. to 1 ft. the wrought-iron plate in fig. 120. Reduce the figure to the equivalent triangle; determine the area of this triangle graphically. State the weight of the plate, given that wrought iron $\frac{3}{8}$ in. thick weighs 15 lb. per square foot.

3. Draw a quadrilateral, using the same lettering as in fig. 112, to the following sizes: AE = 10$\frac{1}{2}$ in., EF = 8$\frac{1}{2}$ in., FD = 1 ft. 0 in., CE = 1 ft. 4 in., FB = 10 in. Scale 1$\frac{1}{2}$ in. = 1 ft. Find its area by the method given

on p. 48. Also reduce it to an equivalent triangle, and find the area. Compare the results.

4. Fig. 121 represents the plan of a piece of level ground, as drawn (or as it is termed " plotted ") from the notes in a surveyor's notebook.

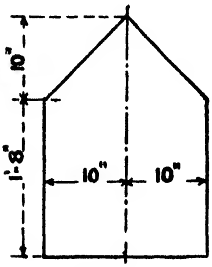


Fig. 119

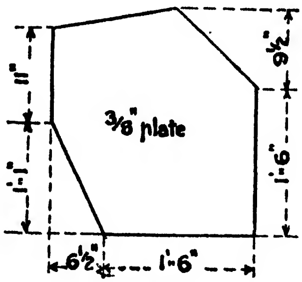


Fig. 120

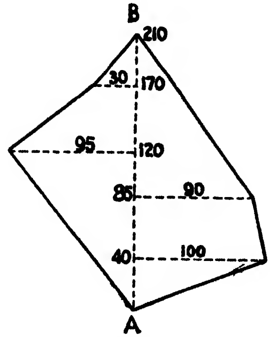


Fig. 121

Note.—Distances along the line AB, which is called the base line of the survey, are entered in the central column, and perpendicular distances

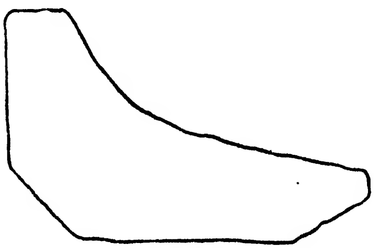


Fig. 122

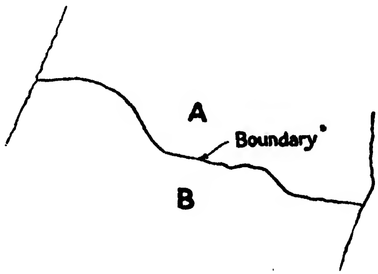


Fig. 123

to right or left of the base line are entered on the corresponding sides of the central column, as shown.

	To B	
	210′	
30′	170′	
95′	120′	
	85′	90′
	40′	100′
	From A	

Draw to a scale of 50 ft. to the inch, and find the area of the plot.

5. Fig. 122 represents an indicator diagram taken from a steam engine. Transfer this to your paper by pricking through, or trace the diagram on tracing paper. Determine the area of the diagram in square inches.

6. Fig. 123 represents an irregular boundary line between two adjacent properties A and B. The owners agree to substitute a straight fence for this irregular boundary, the areas of the holdings to remain the same as before. Draw a "give-and-take" line to represent a straight fence fulfilling these conditions.

CHAPTER VIII

REGULAR POLYGONS

A regular polygon is a plane figure in which all the sides are of the same length and all the interior angles are equal.

A circle called the **circumscribed circle** may be described about any regular polygon to pass through all the corners, and another called the **inscribed circle** may be described inside any regular polygon to touch all the sides at their mid-points.

To describe a regular polygon given the length of side and number of sides. If the number of sides is 3, 4, 6, 8, or 12, these polygons are most conveniently drawn by the use of the set squares.

The following general construction may be used, whatever the number of sides.

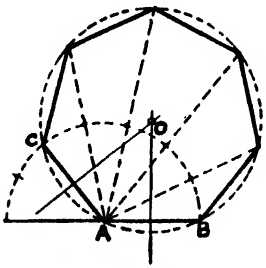


Fig. 124

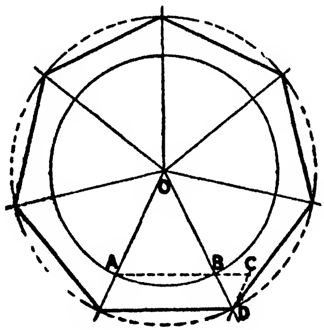


Fig. 125

With one end A of the side AB as centre, and with radius AB, describe a semicircle. Divide the circumference by trial into as many parts as there are to be sides in the polygon. Through A draw lines through the points of division, missing the first, and step the given length of side round between these. In fig. 124 the construction is illustrated for a polygon of 7 sides. Alternatively, having divided the semicircle, bisect AB and AC at right angles (fig. 124) to intersect at O. Draw the circumscribed circle, radius OA, and step the length AB round this.

An alternative, and for practical purposes preferable, construction is indicated in fig. 125.

Draw any circle and divide the circumference into as many parts as there are to be sides in the figure, and draw the radii as indicated. Join two points of division by a chord AB and produce it if necessary, making AC equal to the required length of side. Draw CD parallel to OA. Then OD will be the radius of the circumscribed circle. In fig. 125 the construction is applied to the case of a figure of 7 sides.

In any regular polygon the sum of the interior angles together with four right angles is equal to twice as many right angles as the figure has sides.

In the case of polygons having an even number of sides, the size is frequently specified by giving the dimension " **over the flats** ", i.e. the distance apart of two parallel sides. This is evidently equal to the diameter of the **inscribed circle**. When the size over flats is given, describe a circle of the given diameter, divide the circumference into as many parts as there are to be sides in the figure, and draw tangents through the points of division.

Alternatively, in polygons having an even number of sides the size may be specified by giving the dimension " **over corners** ", i.e. the distance apart of two opposite angles. This is evidently equal to the diameter of the **circumscribed circle.**

When the size over corners is given, describe a circle of the given diameter, divide the circumference into as many parts as there are to be sides in the figure, and join the points of division.

EXERCISES

1. Set out the pentagonal star pattern suitable for an inlay shown in fig. 126, making the diameter of the circumscribed circle $2\frac{1}{2}$ in.

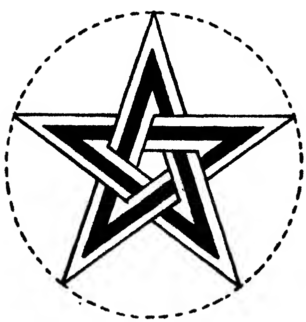


Fig. 126

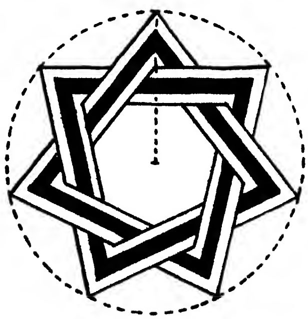


Fig. 127

2. Set out the interlaced band pattern shown in fig. 127, making the diameter of the circumscribed circle $2\frac{1}{2}$ in.

3. Whitworth standard sizes for bolt heads and nuts are approximately: Dimension over flats = $1\frac{1}{2}D + \frac{1}{8}$ in., where D is the diameter of the shank of the bolt. Set out (as shown in fig. 128) the two views of hexagonal nuts for bolts 1 in. and $1\frac{1}{4}$ in. diameter respectively.

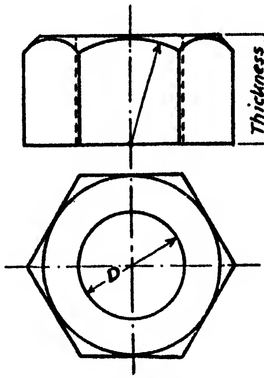


Fig. 128

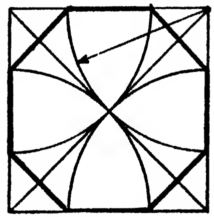


Fig. 129

4. A square balk of Oregon pine, sawn 20 in. square, is to be dressed down to form an octagonal spar. Using the method indicated in fig. 129, draw to a scale of $\frac{1}{8}$ full size a section of the resulting spar, and measure the length of side.

5. What is the length of side of the largest hexagonal spar that can be made from a log 15 in. diameter?

CHAPTER IX

MENSURATION OF THE CIRCLE

The circumference of a circle is equal to $3\frac{1}{7}$ times the diameter, or πd.

Verify this as follows: Cut out a large circle of stiff card, say 5 in. diameter. Measure the circumference by wrapping a piece of thin paper round the curved boundary. Do the same by rolling the circle for one complete revolution along a straight line. Divide the circumference so arrived at by the diameter, that is, find the ratio of the circumference to the diameter. This ratio is the same for any circle and is equal to 3·14159;

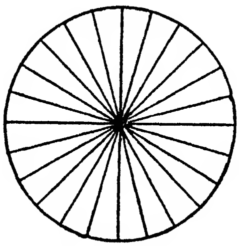


Fig. 130

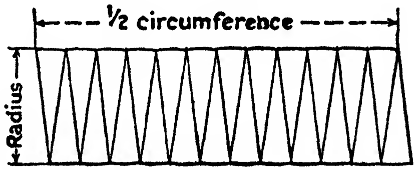


Fig. 131

for practical purposes it may be taken as $3\frac{1}{7}$ or $\frac{22}{7}$, and is usually denoted by the letter π.

The area of a circle is equal to $3\frac{1}{7}$ times the square of the radius, or πr^2.

If any circle is cut along radiating lines into a large number of thin equal sectors, as in fig. 130, it will be seen that each sector is practically an isosceles triangle, but that its base is slightly curved instead of straight. These pieces could be rearranged, as shown in fig. 131, to form a figure which is practically a parallelogram. The greater the number of pieces into which the circle is cut the more nearly will the figure approximate to a straight-sided figure. The base of the parallelogram will equal half the circumference of the circle, its height will equal the radius, and its area will be ($\frac{1}{2}$ circumference) $\times$ (radius), or πr^2.

To Set Out a Straight Line equal in Length to a Circular Arc.

(1) **By "stepping out" with the dividers** (fig. 132).—Open the dividers to some small amount chosen at random—say about $\frac{1}{4}$ in. or so—and step this along the arc AB as many times as it will go. Without altering the dividers, step the distance the same number of times along a straight line CD. Generally in stepping along the curved line a remainder less than a complete "step" will have been left. Measure this with the dividers, and transfer it to the end of the distance stepped out along the straight line. The total distance along CD will now be very nearly equal to the length of the arc.

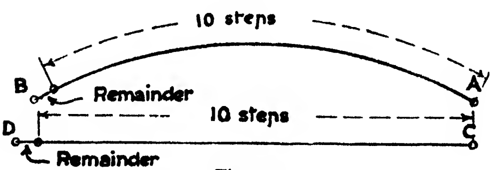


Fig. 132

(2) **By construction** (fig. 133).—Bisect the arc at P and draw the tangent at that point parallel to the chord AB. Join AP and produce to O, making OP equal to half AP. With O as centre, and radius OA, describe an arc cutting the tangent through P at Q. The line PQ will be equal to the arc PA, that is to half the length of the arc APB.

This method gives a very close approximation when the whole arc APB is not more than $\frac{1}{3}$ of a circle.

To Find the Area of a Sector of a Circle (fig. 134).—By the construction above set out PQ equal to half the arc AB, then the rectangle OPQR will be equal in area to the sector APBO.

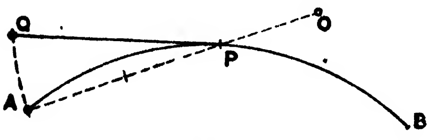


Fig. 133

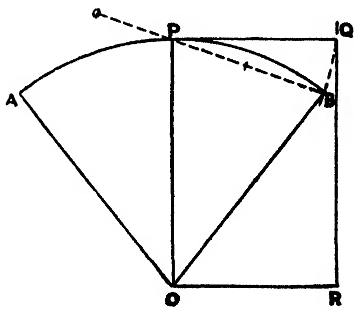


Fig. 134

To Find the Area of a Sector, given the Radius and Angle of Sector.—Multiply the area of the whole circle, of which the sector is a part, by the number of degrees in the angle, and divide by 360. Or, if r is the radius, and N the number of degrees in the angle, the area $= \pi r^2 \cdot \frac{N}{360}$.

To Find the Length of Arc, given the Radius and Angle of Sector.—Multiply the circumference of the whole circle, of which the

sector is a part, by the number of degrees in the angle, and divide by 360; or length of arc $= 2\pi r \dfrac{N}{360}$.

To Set Out a Line Equal in Length to a Semicircle (fig. 135).—Draw the diameter PQ and the tangent at Q. Draw OR, making the angle ORQ 60°. From R measure RS, equal to three times the radius of the circle. Join S to P, then PS is a close approximation to half the circumference of the circle.

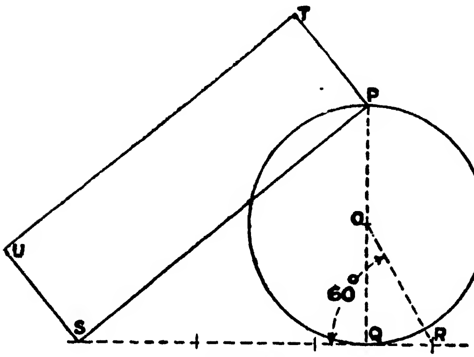


Fig. 135

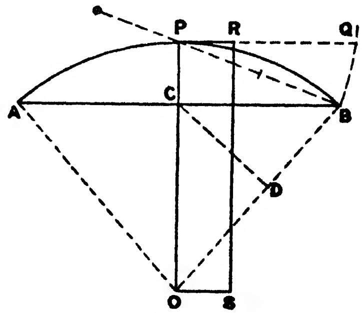


Fig. 136

To Draw a Rectangle Equal in Area to a Circle (fig. 135).—Set out as above a line such as PS, equal to the semi-circumference, and on it describe a rectangle PTUS, making PT equal to the radius. This rectangle will be equal in area to the circle.

To Find the Area of a Segment (fig. 136).—Set out PQ by construction equal to half the arc APB. Draw CD perpendicular to BO, and from Q mark off QR equal to CD. The rectangle OPRS is equal in area to the segment ACBP.

EXERCISES

1. Draw a quadrant of a circle, radius 4 in. Find the length of arc (*a*) by stepping out with the dividers, (*b*) by construction, writing down the length obtained in each case, checking the results by calculation.

2. On a chord of a circle 3 in. long describe an arc of radius 2·1 in., and determine the length of this arc by construction.

3. The length of a circular arc may be obtained from the formula:

$L = \dfrac{8B - A}{3}$, where L = length of arc,
A = length of the chord of the whole arc,
B = length of the chord of half the arc,
all measured in the same units.

Set out an arc of 5 in. radius standing on a chord 6 in. long. Measure the chord of half the arc, and obtain the length of arc by the use of the given formula. Check by construction.

4. Two shafts in a machine shop are 6 ft. apart, and carry pulleys 18 in. diameter (fig. 137). What length of belting would be required to drive one shaft from the other if the shafts are required to rotate in the same direction? The sag of the belting, its thickness, and the allowance for lapping the belt at the junction may be neglected.

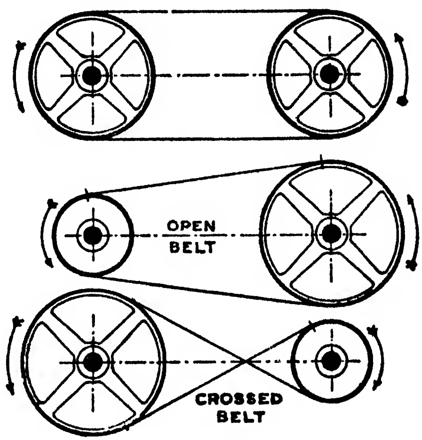


Fig. 137

5. What would be the length of belt required in the foregoing question if the shafts were required to rotate in opposite directions, i.e. if the belt were crossed?

6. Set out and measure the length of leather belting required to connect two pulleys 24 in. and 14 in. diameter respectively, the distance apart of the shafts being 4 ft. 6 in. Scale 1 in. = 1 ft. Assume that an "open" belt is used, and allow 8 in. for lapping (fig. 137).

7. What would be the length of a "crossed" belt connecting two pulleys 30 in. and 20 in. diameter, the distance apart of the shafts being 5 ft. 6 in. and 10 in. allowed for lapping?

Note.—In Exercises 4 to 7 the pulley arms need not be drawn, only such lines being set out as are necessary to arrive at the lengths of belts.

8. By construction set out a rectangle equal in area to a sector of a circle 4 in. radius, the angle of the sector being 60°, and calculate its area in square inches.

9. By construction set out a rectangle equal in area to a segment of a circle, the chord of the segment being $3\frac{1}{2}$ in. long and the radius of the circle $2\frac{1}{2}$ in., and calculate its area.

CHAPTER X

SIMPLE SOLIDS—DEVELOPMENTS

Simple Solid Forms

Various forms of solids are indicated in the small inset diagrams to figs. 138 to 156, and will serve to illustrate the following definitions:

Axis.—The line joining the centres of the parallel ends of a prism or cylinder, or joining the centre of the base to the apex in a pyramid or cone.

Right Solid.—One in which the axis is perpendicular to the end faces or base.

Oblique Solid.—One in which the axis is inclined to the end faces or base.

Height or Altitude.—The perpendicular distance between the end faces or between the apex and base.

A pyramid or cone is said to be **truncated** when a portion of the top is removed by a plane cut. The lower portion standing on the base is called a **frustum** of the pyramid or cone as the case may be.

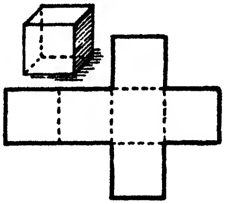
It will be seen that in a right prism the side faces are all rectangles, in an oblique prism the side faces may be parallelograms or rectangles. In a right pyramid all the triangular faces are isosceles, in an oblique pyramid they may be of any shape.

Before proceeding to draw the projections of these solids it would be well for students to make models of some of them for themselves, the time spent being well repaid by the knowledge thus acquired as to the properties of the solids. The name and dimensions of each solid should be neatly written on the base.

When the surface of a solid is opened out or flattened, it is said to be **developed.** Figs. 146 and 147 will make clear the operation of developing the surface of a right cylinder and cone respectively.

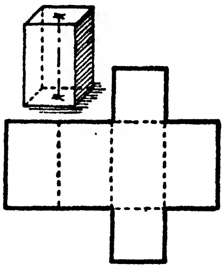
To enable the models to be made the development of each is given in figs. 138 to 156.

White card or " Bristol board " should be used, the form of the development being set out directly upon it with sharp, hard pencil. Thick card



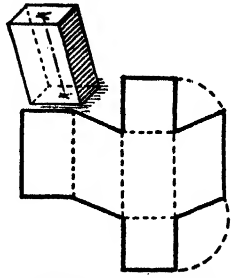
CUBE

Fig. 138



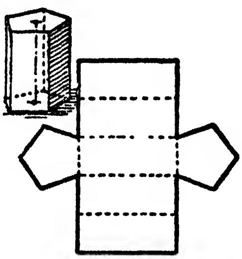
SQUARE PRISM

Fig. 139



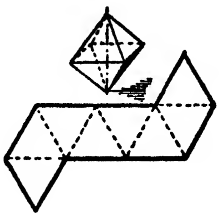
OBLIQUE SQUARE PRISM

Fig. 140



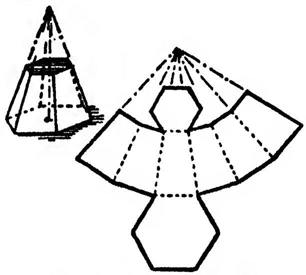
PENTAGONAL PRISM

Fig. 141



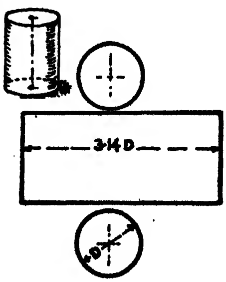
OCTAHEDRON

Fig. 142



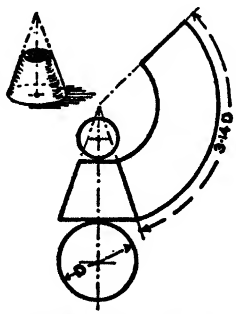
TRUNCATED HEXAGONAL PYRAMID

Fig. 143



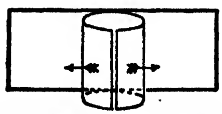
CYLINDER

Fig. 144.



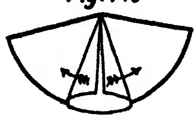
TRUNCATED CONE

Fig. 145



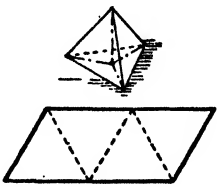
CYLINDRICAL DEVELOPMENT

Fig. 146



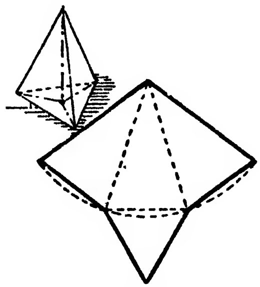
CONICAL DEVELOPMENT

Fig. 147.



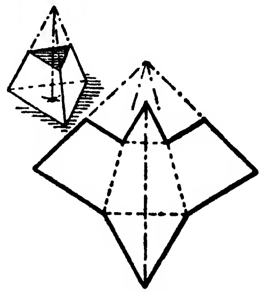
TETRAHEDRON

Fig. 148



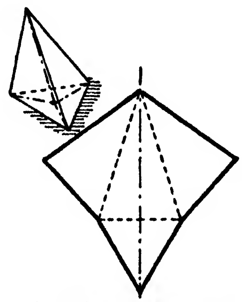
TRIANGULAR PYRAMID

Fig. 149



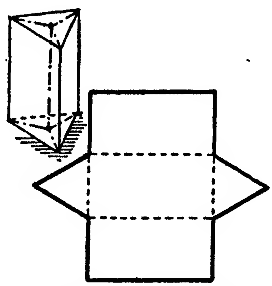
TRUNCATED TRIANGULAR PYRAMID

Fig. 150



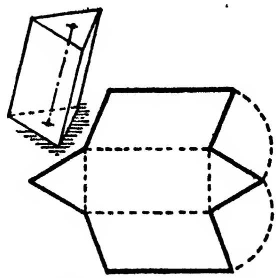
OBLIQUE TRIANGULAR PYRAMID

Fig. 151



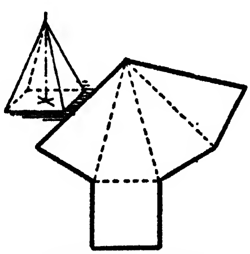
TRIANGULAR PRISM

Fig. 152



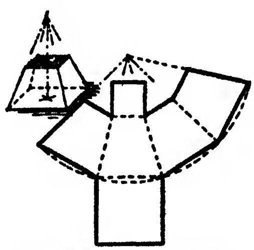
OBLIQUE TRIANGULAR PRISM

Fig 153



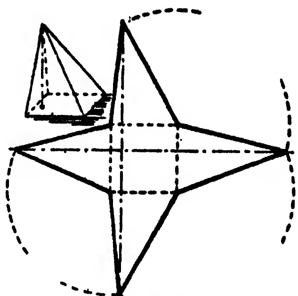
SQUARE PYRAMID

Fig. 154



TRUNCATED SQUARE PYRAMID

Fig 155.



OBLIQUE SQUARE PYRAMID

Fig. 156

should be avoided as producing clumsy models not true to shape. Cut round the outline with a very sharp knife, using a hard clean surface to cut on, such as glass or slate. Make light cuts (not more than half-way through the card) at the dotted lines, and bend the card carefully at these, keeping the cuts to the outside. Make the joints with paper strips, say $\frac{3}{8}$ in. wide, bent to L shape, cut to lengths as required, and fasten with

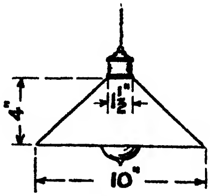


Fig. 157

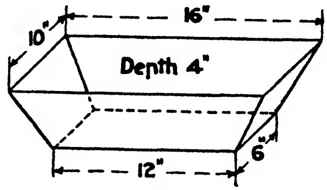


Fig. 158

strong gum. These may be put on the outside or inside (the latter being neater but more difficult), and trimmed to match at the edges. For cylindrical and conical forms, small V-shaped pieces should be nicked out of the jointing strips to enable them to be bent to circular form.

EXERCISES

1. You are required to make a paper shade for the conical portion of the electric-light fitting shown in fig. 157. Set this out $\frac{1}{4}$ full size, allowing a lap of $\frac{1}{2}$ in. to permit of joining up. Do this full size for an actual example if available, and fit it in position.

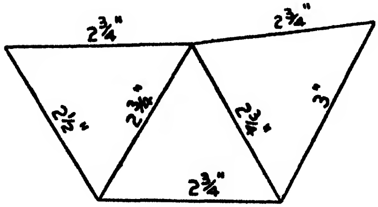


Fig. 159

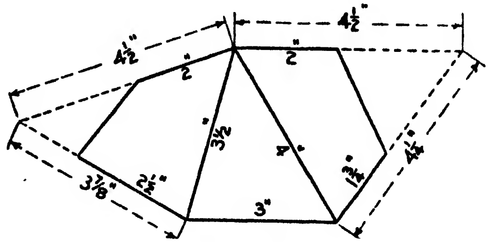


Fig. 160

2. Fig. 158 gives the inside dimensions of a wooden tray to be fitted with a zinc lining. Set out the development of the zinc—no laps need be allowed. Scale $\frac{1}{4}$ full size.

3. The base of an irregular pyramid is given in fig. 159, together with the development of two of the triangular faces. Set out the shape of the third face.

4. The base of a truncated pyramid is given in fig. 160, together with the development of two faces thereof. Set out the shape of the third face. Test your work by cutting out the development of the base and three faces and folding to show the shape of the solid.

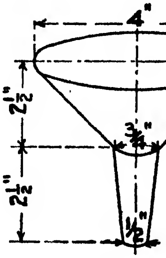


Fig. 161

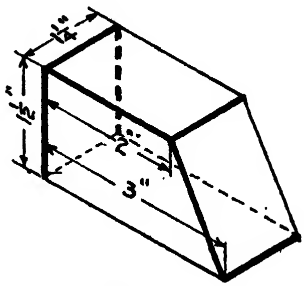
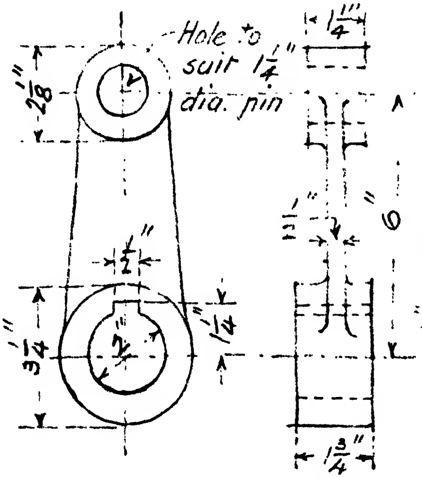


Fig. 162

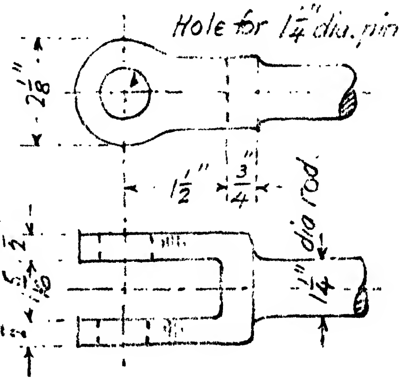
5. Set out the shapes of the templates from which the two pieces of tin required to make the funnel shown in fig. 161 could be cut, neglecting laps.

6. A closed box of the sizes shown in fig. 162 is to be made from a shaped sheet of metal in one piece. Draw full size a development assuming that bends are made at the thin lines and seams at the thick ones.



FORGED W.T I. LEVER

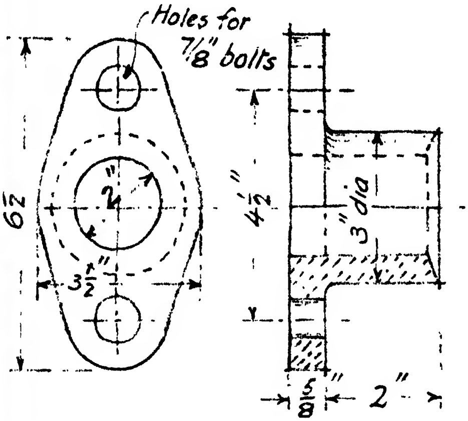
FIG. 163



— FORKED END —

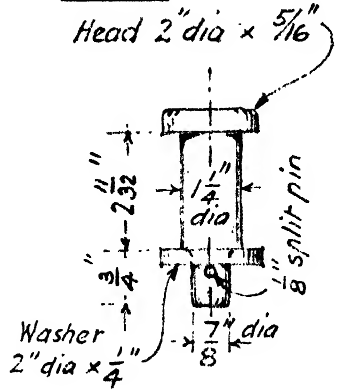
— OF ROD —

FIG 164



GUN-METAL GLAND

FIG 165



— TURNED —

— PIN —

FIG 166

CHAPTER XI

SKETCHING

Hand sketching is an important but frequently neglected branch of practical drawing. In most cases considerable practice will be necessary before any degree of facility is acquired, but the ability to make good sketches is of such importance as to justify time spent in developing it.

A correct eye and a free hand must be cultivated. Outlines should be lightly put in at first, and finished by lining in with firm lines without lifting the pencil from the paper too frequently. Those which are mutually parallel or perpendicular in the object sketched should be reasonably so in the sketch, and while it would not be expected that the sketch would scale correctly, care should be exercised in maintaining the proportions of the object as far as this can be attained without actual measurement on the sketch.

To assist in the development of a correct eye, exercises should be practised involving the estimation of distances and dimensions by eye, i.e. by judging the dimensions without actual measurement.

The following examples will be found to provide good practice. Where the word " sketch " is used, freehand unassisted by instruments is intended. A soft pencil should be used.

1. Sketch (each in one stroke) horizontal lines 1 in., 2 in., 3 in., and 4 in. long.

2. Sketch (each in one stroke) vertical lines $1\frac{1}{2}$ in., 2 in., $2\frac{1}{4}$ in., $3\frac{3}{4}$ in. long.

3. Draw a number of straight lines, and divide them by estimation into 3, 4, 5, 8, 9, 12, &c., parts.

Note.—In subdividing lengths factorial methods will be found to produce the best results; thus, to divide into 12 parts—since $12 = 2 \times 2 \times 3$—first bisect by estimation, bisect again to give quarters, and trisect these to give twelfths.

4. By estimation mark off on a line AB a point C $\frac{2}{3}$ of the length from the end A.

5. From the mid-point of a line CD sketch a line perpendicular thereto and half as long.

6. From the end of a line drawn at random sketch a line perpendicular thereto and $\frac{5}{8}$ the length.

7. As far as can be judged, without use of the scales, sketch in various positions lines representing, to a scale of 3 in. to 1 ft. 0 in. ($\frac{1}{4}$ full size), the following dimensions: 5 in., 8 in., $10\frac{1}{2}$ in., 1 ft. 2 in., 1 ft. 6 in., &c.

8. Estimate and sketch lines representing, to a scale of $\frac{1}{8}$ full size ($1\frac{1}{2}$ in. to 1 ft. 0 in.), the following dimensions: 8 in., 1 ft. 3 in., 2 ft. 1 in., &c.

9. Estimate and sketch lines representing, to a scale of 1 in. to 1 ft. 0 in. ($\frac{1}{12}$ full size), the following dimensions: 2 ft. 3 in., 2 ft. $7\frac{1}{2}$ in., 3 ft. 4 in., &c.

The lines chosen should be in various positions, and it will be noted that the estimation of lengths is generally less accurate as the lines approach a vertical position, the common tendency being to underestimate these.

Estimation of Angles and Direction.—This must also be practised, and the following and similar examples will be found helpful:—

10. Draw a number of lines in various directions, and sketch lines parallel to these at distances of $\frac{1}{2}$ in., $\frac{3}{4}$ in., 1 in., &c., away.

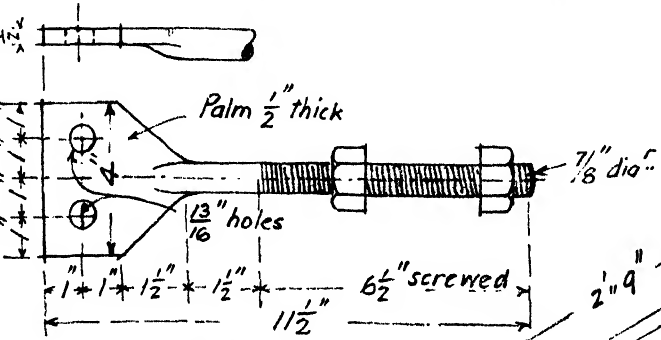
11. Sketch pairs of lines making angles of 30°, 45°, and 60° with each other.

12. Sketch lines representing the relative positions of the hands of a clock at 2 o'clock, 3.30, 9.35, and so on.

13. Sketch two lines at right angles to each other to represent the N. and S., E. and W. compass directions. Sketch in other lines representing compass bearings of N.W., N.N.E., E.S.E., N.E. by E., N.E. by N., &c.

Obviously the best practice in sketching is to be obtained by actually measuring with rule and callipers simple details of actual work. In most institutions models and examples of engineering and building details are available, and these should be measured, hand sketched, and scale drawings prepared from the sketches. Although its use is best deferred until proficiency has been acquired in sketching on plain paper, squared paper will be found of great assistance in sketching, the ruled lines serving as a guide in the drawing of straight lines and in maintaining proper proportion. Since sketching for the purpose here used is not intended as a test of freehand drawing, when plain paper is used no objection can be taken to using the edge of the rule for putting in the longer straight lines. Figs. 163 to 169 show examples illustrative of the manner in which sketches from measured details should be prepared. These may be copied by hand and scale drawings prepared from the sketches.

Where the objects sketched are suitable, it is often convenient to combine all the information on one view, and use the methods of pictorial or isometric projection explained in later chapters.



W.T.I. PALM BOLT

FIG 167

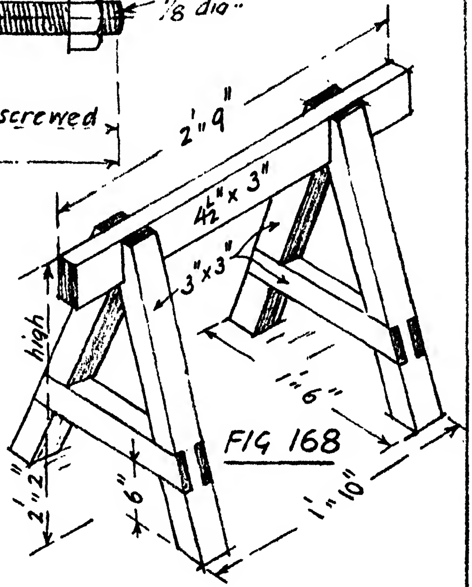


FIG 168

CARPENTERS HORSE

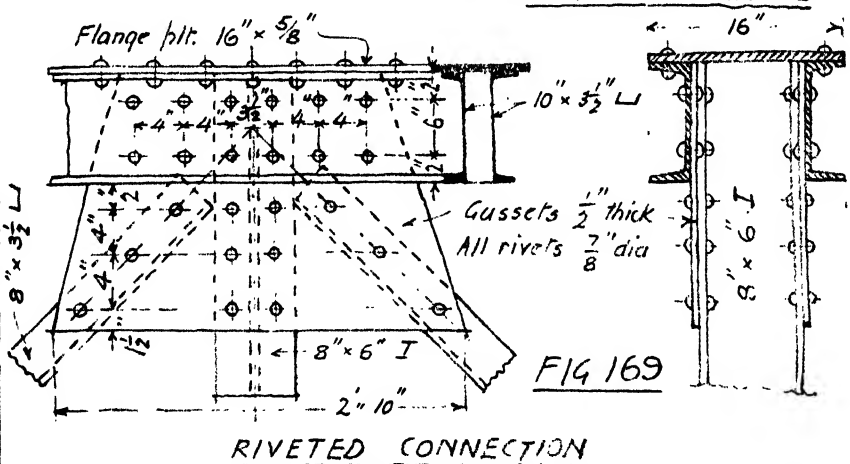


FIG 169

RIVETED CONNECTION

CHAPTER XII

PICTORIAL PROJECTION

In attempting to draw or sketch an object in such a manner as to convey as clear an idea as possible of its shape, the first and probably most natural procedure that would occur to one not acquainted with the conventions of geometry would be to stand in front of the object and to sketch a picture of what is presented to the eye. The resulting sketch would approximate to what would be obtained if a camera were substituted for the eye and a photograph taken. In such a drawing one of the chief difficulties in making a convincing representation is caused by the well-known perspective effect, due to which parallel lines receding from the eye appear to converge towards a distant point, and distances which we know to be equal have to be represented by lengths which vary according as they are close to or far from the eye.

For technical purposes this effect serves no useful purpose, but rather the reverse, as it renders it impossible by direct reference to the drawing to arrive at the dimensions of the object, and it is found much more convenient to have recourse to methods by which we can give an approximately just impression of the appearance of an object, and still be able to represent sizes by direct measurement.

Three methods are in general use:—

Pictorial or, as it is sometimes called, **Oblique Projection;**
Isometric Projection;
Orthographic Projection.

The examples given in figs. 170, 171 and 173 represent views of the same object drawn to the same scale in each of these methods of projection, and fig. 172 a perspective view.

It will be seen that pictorial and isometric projections are modified forms of perspective and enable all the necessary information to be conveyed on one view, while in orthographic projection more than one view is required (fig. 173).

Pictorial Projection.—Strictly speaking, pictorial projection is not

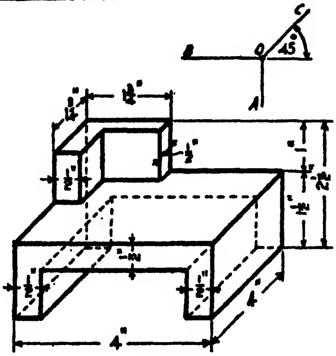


Fig. 170
PICTORIAL PROJECTION

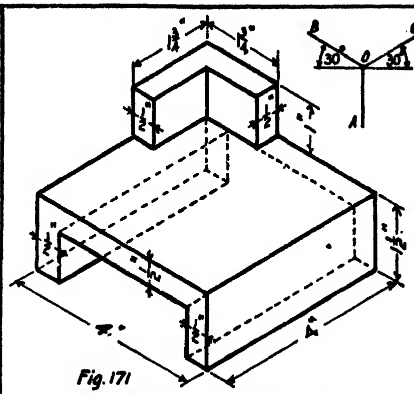


Fig. 171
ISOMETRIC PROJECTION

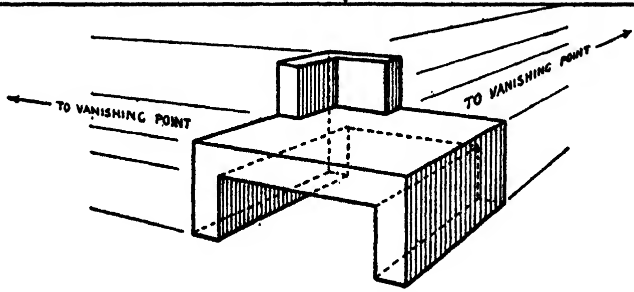
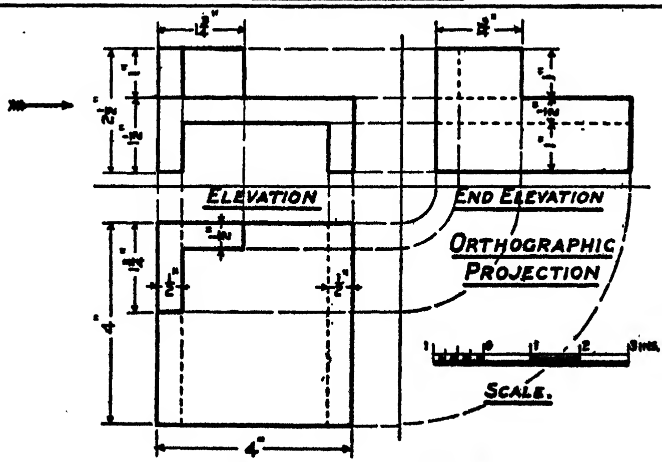


Fig. 172 PERSPECTIVE VIEW



PLAN Fig 173

a true projection, but custom sanctions its use, and it is frequently employed by joiners and in the building trades. The following are the principles involved:

All lines which are parallel in the object are parallel in the projection.

One face of the object is assumed to be at right angles to the line of sight, and is represented true to shape and scale.

Edges at right angles to this face are represented by lines drawn with the set square, and to avoid apparent distortion of the object are set out to a reduced scale. Although not invariable, the use of an angle of 45° for edges perpendicular to the front face, and a scale of half that employed for the front face, will produce good figures, and is recommended. Other angles and reductions may be used for these edges where the conditions

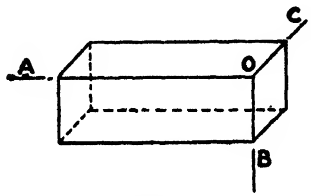


Fig. 174

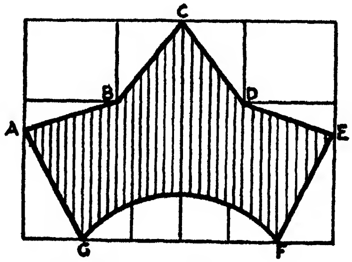


Fig. 175

render it advisable. Thus, in fig. 174, which represents a brick 9 in. × 4½ in. × 3 in. in its " laid " position, the front or " stretcher " face 9 in. × 3 in. is represented true to shape, and the 4½ in. edges at right angles thereto are drawn with the 45° set square. Dimensions parallel to OA and OB are drawn full size, and those parallel to OC are drawn half full size.

This form of projection is more suitable for sketching purposes than for scale drawing, and is best restricted to objects which are mainly rectangular. It often serves a useful purpose when making preliminary sketches of objects preparatory to making scale projections of them.

To Draw the Pictorial Projection of an Irregular Figure.—When a figure has few or none of its edges parallel to the main axes of projection, enclose it in a rectangle having its sides parallel to the main axes. Draw auxiliary lines parallel to the main axes to locate other points on the outline of the figure. The projection of the rectangle and auxiliary lines enable all the points in the projection of the figure to be located.

Example.—The figure ABCDEFG, fig. 175, is enclosed in a rectangle, and auxiliary lines drawn through B and D and to the curved outline FG. Figs. 176 and 177 show the figure in pictorial projection in two positions.

To Draw the Pictorial Projection of a Circle.—Enclose the circle in a square and draw the diagonals. Draw auxiliary lines through the points where the diagonals intersect the circle.

The projection of these lines locates 8 points on the projection of the circle.

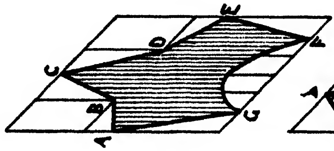


Fig. 176

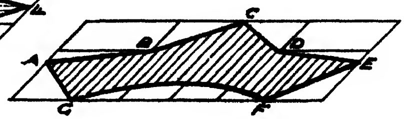


Fig. 177

Alternatively.—Draw two perpendicular diameters. Divide a radius into a convenient number of parts and draw chords of the circle through the points of division. Draw the projection of these diameters and chords, setting out the lengths of the half chords on each side of the diameter to obtain points on the projection of the circle.

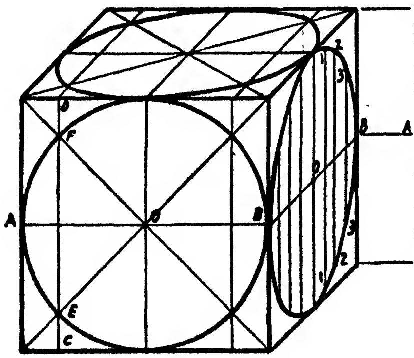


Fig. 178

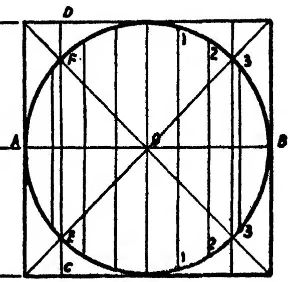


Fig. 179

Example.—Figs. 178 and 179 show tne above methods applied to the pictorial projection of a cube with a circle inscribed in each of the three visible faces.

Note.—The projection of a circle in the plane at right angles to the line of sight will be an equal circle, while that of a circle in any other plane (including the other two main planes) will be an ellipse.

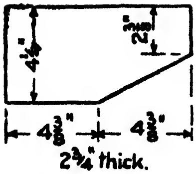


Fig. 180

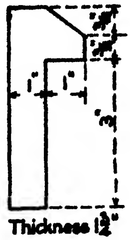


Fig. 181

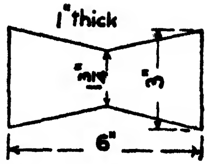


Fig. 182

EXERCISES

Make pictorial projections of the following objects:

1. A right square pyramid 2½ in. high standing on a 2 in. square base.
2. A hexagonal prism, edge of base 1¼ in., length 3 in., (*a*) when standing on end, (*b*) when lying on its side with the end presented to the observer.
3. A right cone standing on a base 2½ in. diameter, height 3 in.

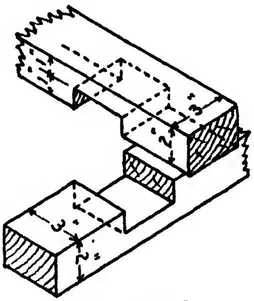


Fig. 183

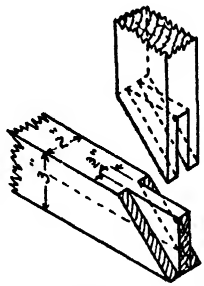


Fig. 184

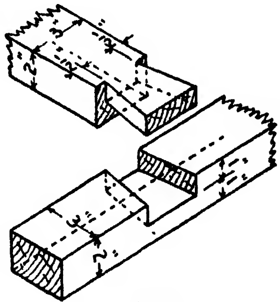


Fig. 185

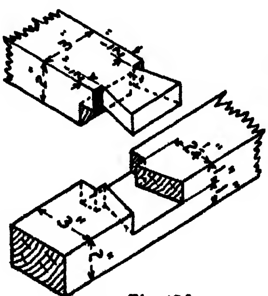


Fig. 186

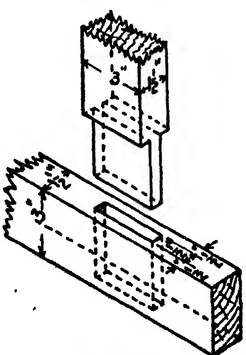


Fig. 187

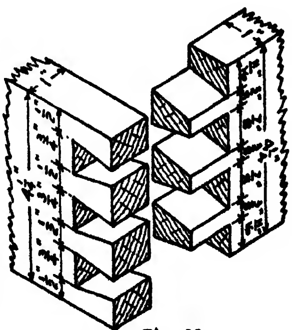


Fig. 188

4. Make a pictorial projection of the "king closer" (a brick cut to the shape indicated for the purpose of obtaining bond in a wall) represented in fig. 180. Scale ½ full size.

5. A hammer-headed steel key is shown in fig. 181. Draw a pictorial projection, the side of the key being presented to the observer.

6. A slate cramp, as used by a mason in jointing two stones, is shown in fig. 182. Make a pictorial projection.

7. Make pictorial projections, ½ full size, of the joints in woodwork illustrated (in isometric projection) in figs. 183 to 188, separating the members in a similar manner to that indicated in the figures.

CHAPTER XIII

ISOMETRIC PROJECTION

In pictorial projection it will have been noticed that inconvenience is introduced owing to the necessity of employing two scales to represent the same object. In isometric projection this disadvantage is avoided, and all dimensions parallel to either of the three principal axes are represented to the same scale.

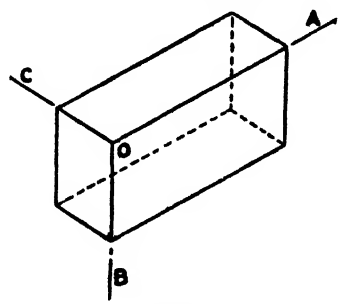


Fig. 189

If a cube or a rectangular prism is so placed that three of the mutually perpendicular edges which meet at a corner are all equally inclined to the line of sight — for example, the edges OA, OB, and OC (fig. 189) — and a projection of the solid made on a plane at right angles to the line of sight, the view obtained would be similar to that shown in the figure, and would be an **isometric projection.**

The projections of the three edges OA, OB, and OC will be equally inclined to each other, i.e. the angles AOB, BOC, and COA will each be 120°, all lines parallel to these edges in the object will be parallel in the projection, and, since the edges are all equally inclined to the plane of projection, equal lengths in the object along the edges OA, OB, and OC will be represented by shorter but equal lengths on the projection. The latter fact is expressed in the name given to this method of projection.

By constructing a scale of reduction in which actual dimensions are represented by their projected lengths, known as an isometric scale, a true isometric projection of a solid can be set out.

To Construct an Isometric Scale.—Draw lines AB, AC, AD (fig. 190) so that AD and AC are inclined at 45° and 30° respectively to AB. Then if actual lengths of the object in the direction of the principal axes are set out on AD, the isometric lengths will be found by projecting them on to AC by lines perpendicular to AB. A true scale drawn on AD can thus be converted into a foreshortened or isometric scale on AC.

As, however, the degree of foreshortening of all lengths parallel to the principal axes is the same, no distortion of the projection is caused by measuring actual lengths parallel to these lines. The resulting projection is similar to the true projection but somewhat larger. In practice this is generally done and the isometric scale seldom used.

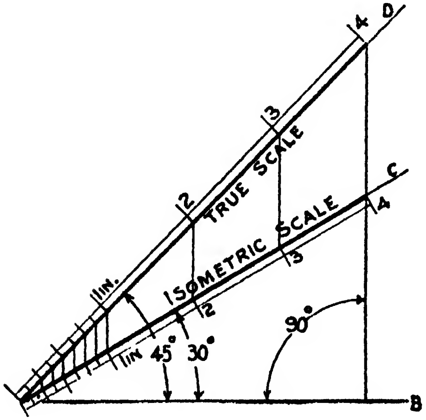


Fig. 190

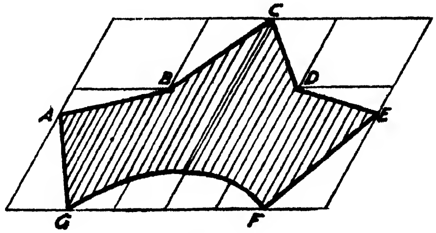


Fig. 191

Thus if fig. 189 represents the projection of a brick $8\frac{3}{4}$ in. × $4\frac{1}{4}$ in. × $2\frac{3}{4}$ in., we should first set out the three lines OA, OB, and OC, using the

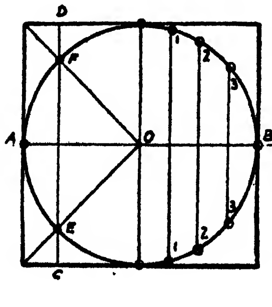


Fig. 192

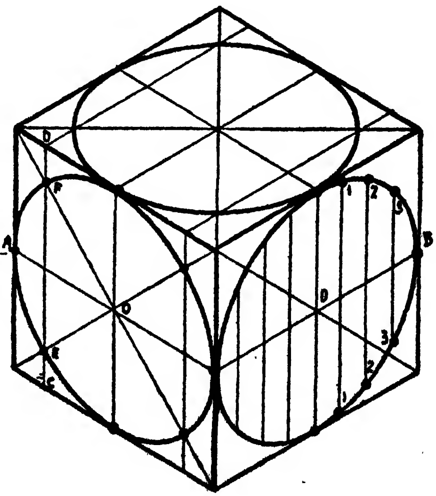


Fig. 193

T square and 30° set square, measure the height of $4\frac{1}{4}$ in. from O to B, the length of $8\frac{3}{4}$ in. from O to A, and the width of $2\frac{3}{4}$ in. from O to C and complete the parallelograms representing the six faces.

To Draw the Isometric Projection of an Irregular Figure.—The method previously described under pictorial projection should be used, the enclosing rectangle and auxiliary lines being projected to locate points on the outline. For example, fig. 191 is an isometric projection of the figure ABCDEFG, fig. 175.

To Draw the Isometric Projection of a Circle.—Use similar methods to those previously described under pictorial projection. For example, figs. 192 and 193 show the isometric projection of a cube with a circle inscribed in each of the three visible faces. Note that circles on any of the three main planes project as similar ellipses.

EXERCISES

Draw isometric projections of the following:

1. A washer plate 6¾ in. square, with a central square hole 1½ in. side. Thickness of plate 1 in. Draw ½ full size.

2. A wedge 12 in. long, 3 in. wide. Height at ends 2¼ in. and ¾ in. respectively. Draw ⅓ full size.

3. A hexagonal prism lying on its side, edge of base 1 in., length of prism 4 in.

4. A "hipped" roof for a building 30 ft. × 20 ft. Height of ridge above eaves 5 ft. 6 in. Scale ⅛ in. = 1 ft.

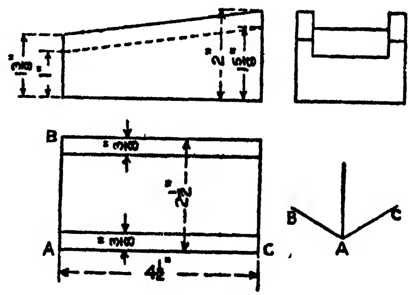


Fig. 194

5. Three views of a steel cotter or wedge are given. Make an isometric projection, placing the edges AB and AC of the base in the positions indicated in the accompanying diagram (fig. 194).

6. Pictorial projections of a gib and cotter are given in figs. 195 and 196. Draw to a scale of ½ full size an isometric projection of each.

7. Fig. 35 gives two views of a terra-cotta ridge tile with the scale to which these are drawn. Draw these views to a scale of 3 in. to 1 ft. Draw an isometric projection of the tile to the same scale.

8. Fig. 197 represents a "haunch tenon-and-mortise" joint for a timber frame. Draw ½ full size an isometric projection, showing the tenon withdrawn from the mortise.

9. Fig. 198 gives two views of a "birdsmouth" joint between a sloping rafter and a wall plate. Draw an isometric projection ½ full size.

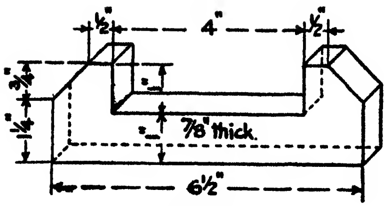


Fig. 195

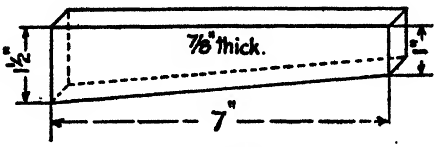


Fig. 196

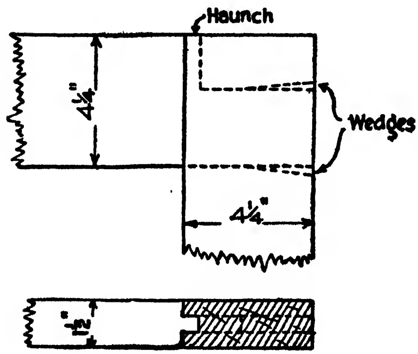


Fig. 197

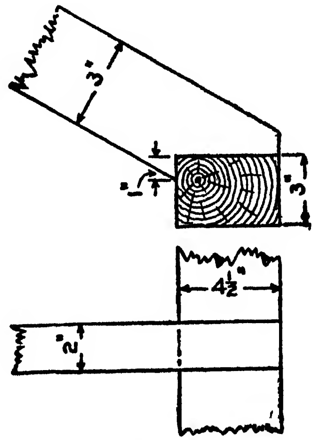


Fig. 198

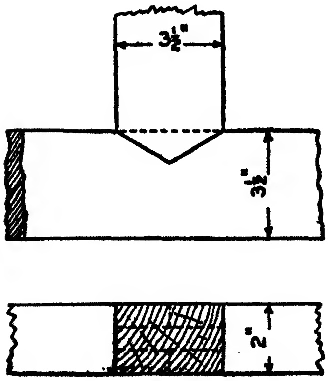


Fig. 199

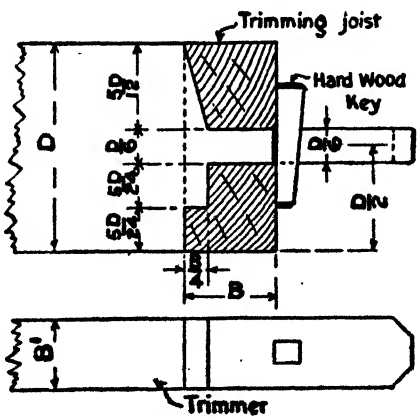


Fig. 200

10. Make an isometric projection of the " bridle " joint shown in fig. 199 to ½ full size, separating the two members so as to show the construction clearly.

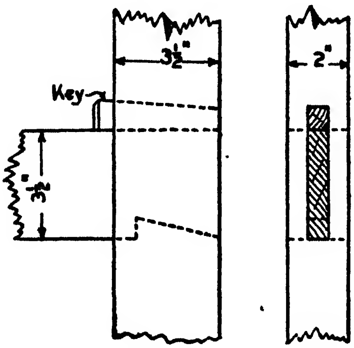


Fig. 201

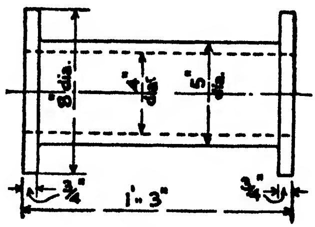


Fig. 202

11. The correct proportions for a " tusk-tenon " joint in a floor between a trimmer and a trimming joist are given in fig. 200. Draw the figure ¼ full size when the trimmer is 10 in. × 3½ in. and trimming joist 10 in. ×

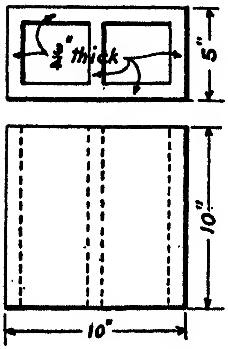


Fig. 203

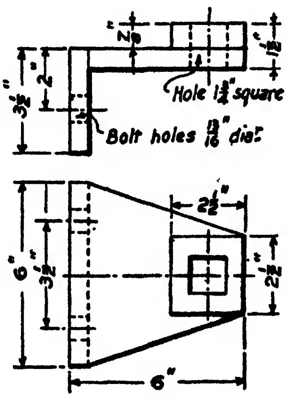


Fig. 204

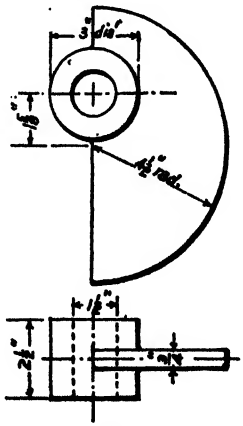


Fig. 205

4 in., i.e. when D = 10 in., B = 4 in., and B′ = 3½ in. Add an isometric projection of the end of the trimmer, supposing it withdrawn from place.

12. A " dovetail mortise-and-tenon " joint is shown in fig. 201. Make an isometric projection ½ full size, separating the two members.

Draw isometric projections of:

13. A circular plate 3 in. diameter, ½ in. thick.

14. A right circular cone base 3 in. diameter, height 3½ in.

15. An uncovered cylindrical box 3½ in. diameter, the side and bottom being ¼ in. thick.

16. The length of flanged cast-iron pipe shown in fig. 202. Scale ¼ full size.

17. The hollow tile shown in fig. 203. Scale ¼ full size.

18. The cast-iron bracket shown in fig. 204. Scale ½ full size.

19. The cam plate shown in fig. 205. Scale ½ full size.

CHAPTER XIV

ORTHOGRAPHIC PROJECTION

The methods of pictorial and isometric projection given in the foregoing chapters are limited in their application and usefulness, and while suitable for the representation of simple forms and those in which the majority of the bounding lines are mutually perpendicular, become complicated when applied to more elaborate forms. It will also be appreciated that to use these methods for the general view of a machine, or for the drawing of a building, for examples, would produce a result which, though pictorial, would be quite unsuitable as a working drawing.

The method universally adopted for the preparation of working drawings is illustrated pictorially in fig. 206, and is known as **orthographic projection.** Let us imagine the object to be represented (in this case the same object as is represented in figs. 170 to 173) placed in position in the space between two flat surfaces or planes at right angles to each other, one of them vertical (the **Vertical Plane,** V.P.), and the other horizontal (the **Horizontal Plane,** H.P.).

Through all the points of the solid let perpendiculars be drawn to the horizontal plane. When the points where these perpendiculars (or **projectors,** as they are called) meet the H.P. are joined, they form what is known as a "**projection**" of the solid on the horizontal plane. It will be seen that the figure so drawn presents the view of the solid which would be obtained by looking vertically down on it from above. Such a view is called a **Plan.**

Again, if perpendiculars (or projectors) are drawn through the points of the solid to the vertical plane, and the feet of these perpendiculars joined, we obtain a projection of the solid on the vertical plane, and the figure so obtained presents the view of the solid which would be obtained by looking in a horizontal direction from the front. Such a view is called an **Elevation.**

If another vertical plane V_1P_1 perpendicular to both H.P. and V.P. is introduced as shown in the figure, and in a similar manner a projection made on this plane V_1P_1, this would present the view obtained by looking

in a horizontal direction from the side in the direction indicated by the arrow. Such a view is called an **End Elevation.**

In order to represent these projections on the flat surface of a sheet of paper, we imagine the planes of projection to be opened out flat, hinging

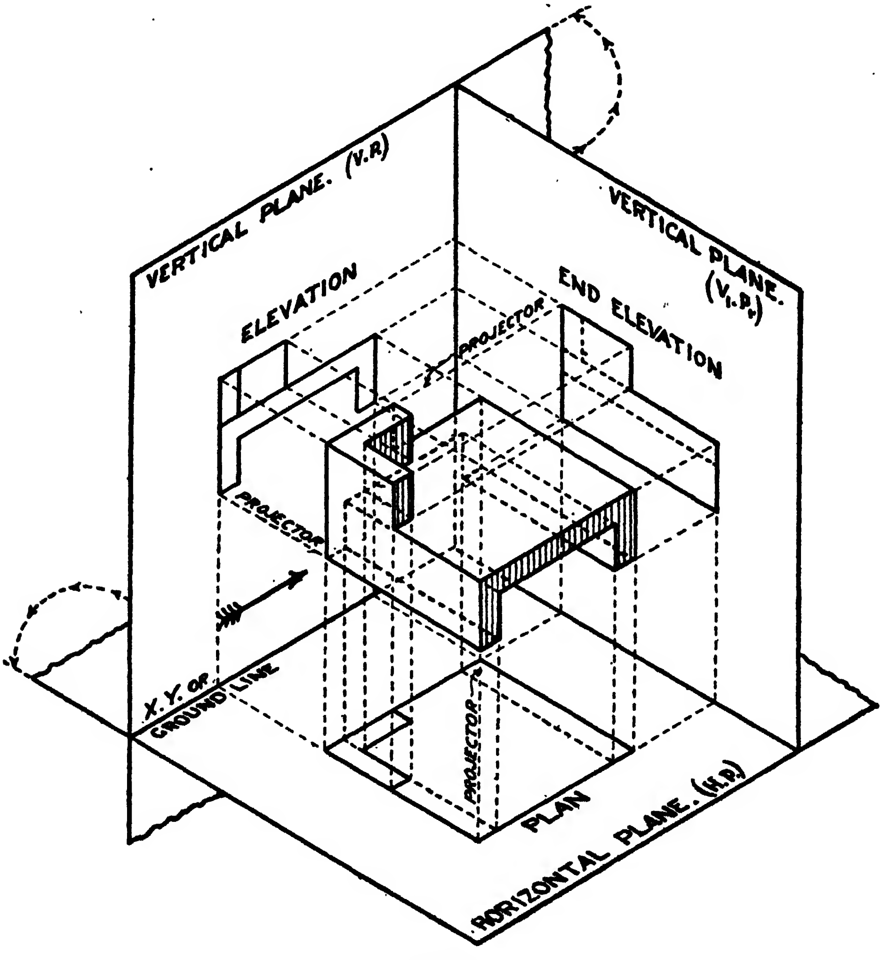


Fig. 206

about the lines in which H.P. and V.P. and V.P. and V_1P_1 intersect. The resulting drawing would then appear as shown in fig. 173. The elevation is directly above the plan and projected therefrom, and the end elevation projected horizontally from the elevation.

EXERCISES

1. Fig. 207 represents in isometric projection a cast-iron bracket. Draw to $\frac{1}{2}$ full size a plan, elevation, and end view of the bracket, and dimension the views.

2. Fig. 208 represents a cast-iron angle block. Draw to $\frac{1}{2}$ full size a plan, elevation, and end elevation looking in the direction of the arrows A, B, and C respectively, and dimension the views.

3. Fig. 209 represents in isometric projection a hanger for the support of a steam pipe. Draw this projection and also draw plan, elevation, and side elevation looking in the direction of the arrows A, B, and C respectively. Scale 3 in. to 1 ft.

4. Fig. 210 shows a pictorial projection of a cast-iron cable support. Draw plan, elevation, and side elevation $\frac{1}{2}$ full size.

5. Fig. 211 represents a bracket bearing for a spindle. Draw $\frac{1}{2}$ full size the three views obtained when looking in the direction of the arrows A, B, and C respectively.

6. Fig. 212 shows four courses of brickwork in an 18-in. square pier. Draw to $1\frac{1}{2}$ in. to 1 ft. an elevation and the plans of two successive courses of brickwork, indicating the joints by simple lines.

7. Draw $\frac{1}{2}$ full size plan and elevation, and an end elevation looking in the direction of the arrow A, of the casting shown in fig. 213, and dimension the views.

8. A plan of a footstep bearing is given in fig. 214, and also a sectional elevation. The outer casing is of cast iron, and the bush of gun metal. Draw a plan and elevation of each of the two portions when separated, dimensioning the views completely.

9. Fig. 215 shows a "quarter bend" for a cast-iron flanged steam pipe. The following list gives the standard dimensions used in practice for steam pressures between 125 and 225 lb. per square inch:

D = Bore of Pipe	D + 3″ = Centre of Flange to Face	R = Radius to Centre of Bend	Diameter of Flange	Thickness of Flange	Thickness of Pipe
4″	7″	$4\frac{3}{4}$″	9″	$1\frac{1}{8}$″	$\frac{11}{16}$″
6″	9″	$6\frac{1}{2}$″	12″	$1\frac{1}{4}$″	$\frac{3}{4}$″

Draw an outside elevation, and project a plan therefrom, for each of the quarter bends given by the above table. Scale 3 in. = 1 ft.

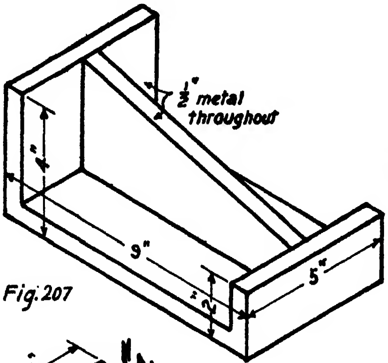


Fig. 207

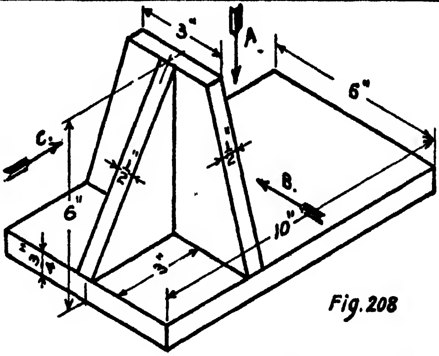


Fig. 208

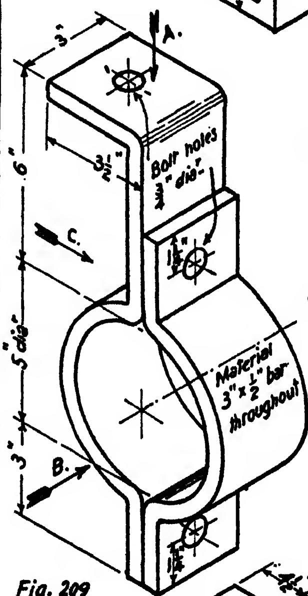


Fig. 209

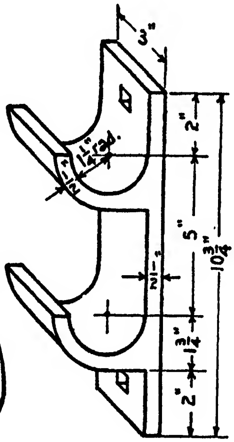


Fig. 210

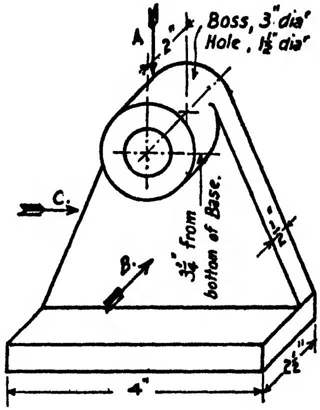


Fig. 211

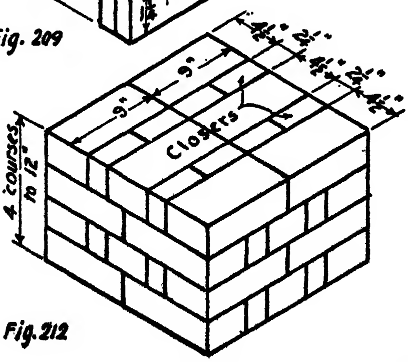


Fig. 212

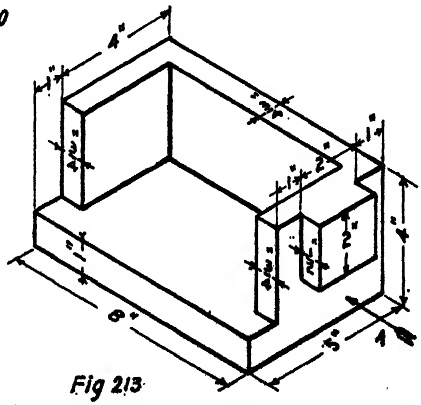


Fig 213

10. Fig. 216 shows a T piece for a cast-iron flanged steam pipe. Draw ¼ full size outside elevation, plan, and end view for 4 in. and 6 in. T pieces respectively, applying the sizes given in the foregoing table.

The objects projected so far in this chapter have been placed with their principal faces parallel with the planes of projection. It is often required to project objects which are inclined to the H.P. or the V.P., or to both. Examples of such projections are shown in figs. 217 to 219.

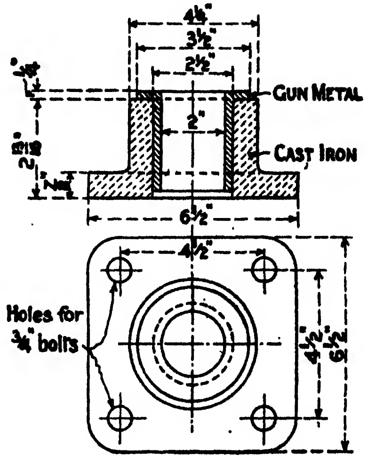


Fig. 214

Fig. 217 gives the projection of a circle perpendicular to V.P. and inclined at an angle A to H.P.

The elevation is a straight line of length equal to the diameter of the circle. Draw a semicircle on this line as diameter. Divide the diameter into an even number of equal parts, and draw ordinates to the semicircle such as 1–7. From the points of division draw projectors, and measure the ordinates 1–7 on the corresponding projectors on each side of a line 0.8 parallel with XY. Draw the curve joining these points to obtain the plan, which is an ellipse.

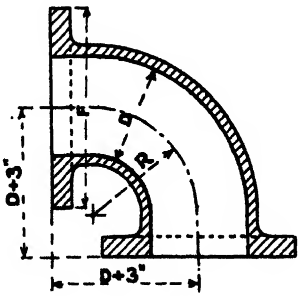


Fig. 215

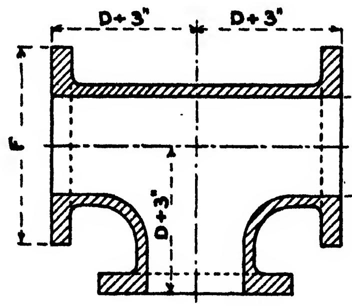


Fig. 216

Fig. 218 gives the projections of a square slab lying on H.P. and having two vertical faces inclined at 30° to V.P.

Draw the square representing the plan of the slab with two edges inclined at 30° to XY. Draw projectors from the corners to meet a line at a height above XY equal to the thickness of the slab, and complete the elevations as shown.

Fig. 219 gives the projections of the same slab as above, placed perpendicular to V.P. with one long edge on H.P. and the two square faces inclined at 55° to H.P.

Draw the rectangle representing the elevation with the two long edges inclined at 55° to XY. Draw projectors from the corners to meet two lines

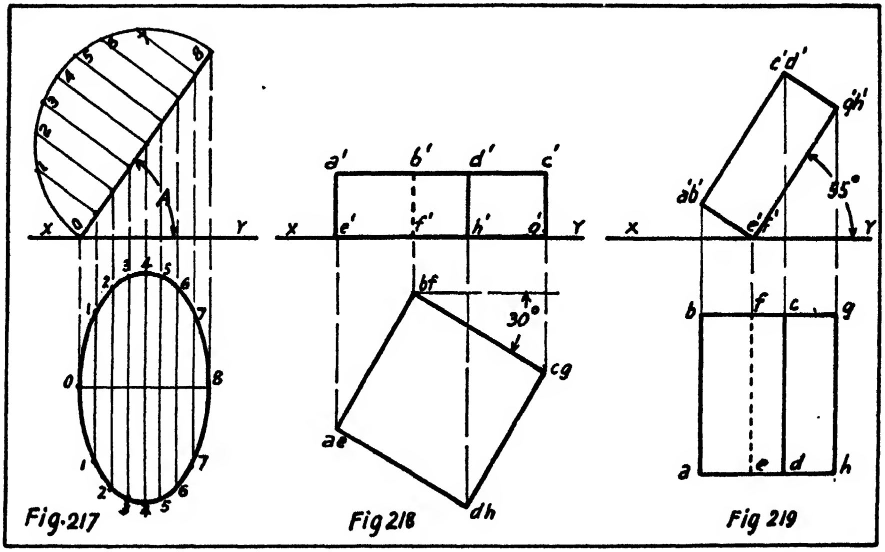


Fig. 217 Fig. 218 Fig. 219

parallel with XY and at a distance apart equal to the side of the square face, and complete the plan as shown.

The models made as described in Chapter X should be set up in various positions between two surfaces representing H.P. and V.P., and plans and elevations drawn to illustrate examples such as the following:—

EXERCISES

Draw projections of:—

11. A tetrahedron, 1½ in. edge, standing on H.P. with one edge of the base perpendicular to V.P.

12. A triangular prism, edges of ends 1½ in., altitude 2½ in., lying with one rectangular face on H.P. and the axis inclined at 35° to V.P.

13. A truncated square pyramid, the square parallel ends 2½ in. and 1¼ in. side respectively, height between ends 1¾ in., placed with one 2½ in.

edge on H.P., the axis parallel with V.P., and the square ends inclined at 45° to H.P.

14. A cylinder, 2 in. diameter, 3 in. long, lying on H.P. with axis parallel with V.P.

15. A cylinder, 2 in. diameter, 3 in. long, lying on H.P. with axis inclined at 40° to V.P.

16. A cone, base $2\frac{1}{2}$ in. diameter, altitude 3 in., lying on its side on H.P. with axis parallel with V.P.

17. A truncated cone, end faces 3 in. and $1\frac{1}{2}$ in. diameter respectively, height between ends 2 in., the axis parallel with V.P. and base inclined at 30° to H.P.

Change of Point of View.—The projections dealt with thus far, i.e. elevation, plan, and end elevation, have all been projected on planes mutually perpendicular, and the objects have been viewed in three directions at right angles to each other; but it frequently occurs, in order that some particular aspect of an object may be more clearly displayed, that a projection is required in a suitable direction inclined to the main planes of projection.

Thus, fig. 220 shows plan and elevation of a rectangular solid tilted at an angle to the horizontal, and we may require to project a **new elevation** when looking at the solid in the direction indicated by the arrow in plan. To obtain this, draw a new ground line X^1Y^1 at right angles to the direction of the arrow, this line representing the position of a new vertical plane on which a new elevation is to be projected. Draw from each of the points on plan projectors at right angles to X^1Y^1, and on these mark above X^1Y^1 the corresponding height of the point above the original XY, transferring the distances with the dividers. The points so obtained are then simply to be joined in proper order to obtain the required projection as shown in the figure.

Or again, fig. 221 shows plan and elevation of a truncated square pyramid, and we may require to project a **new plan** when looking at the solid in the direction indicated by the arrow in elevation. In this case draw a new ground line X_1Y_1 at right angles to the line of view, and from each of the points in the elevation draw projectors at right angles thereto, measuring below X_1Y_1 the corresponding distance of the point below the original XY, and join the points so obtained to produce the required new plan.

Until facility is acquired beginners will do well to letter the points as shown as an aid to identification, the points being projected one at a time, as, if all the projectors are drawn as a first step, in the desire perhaps to save

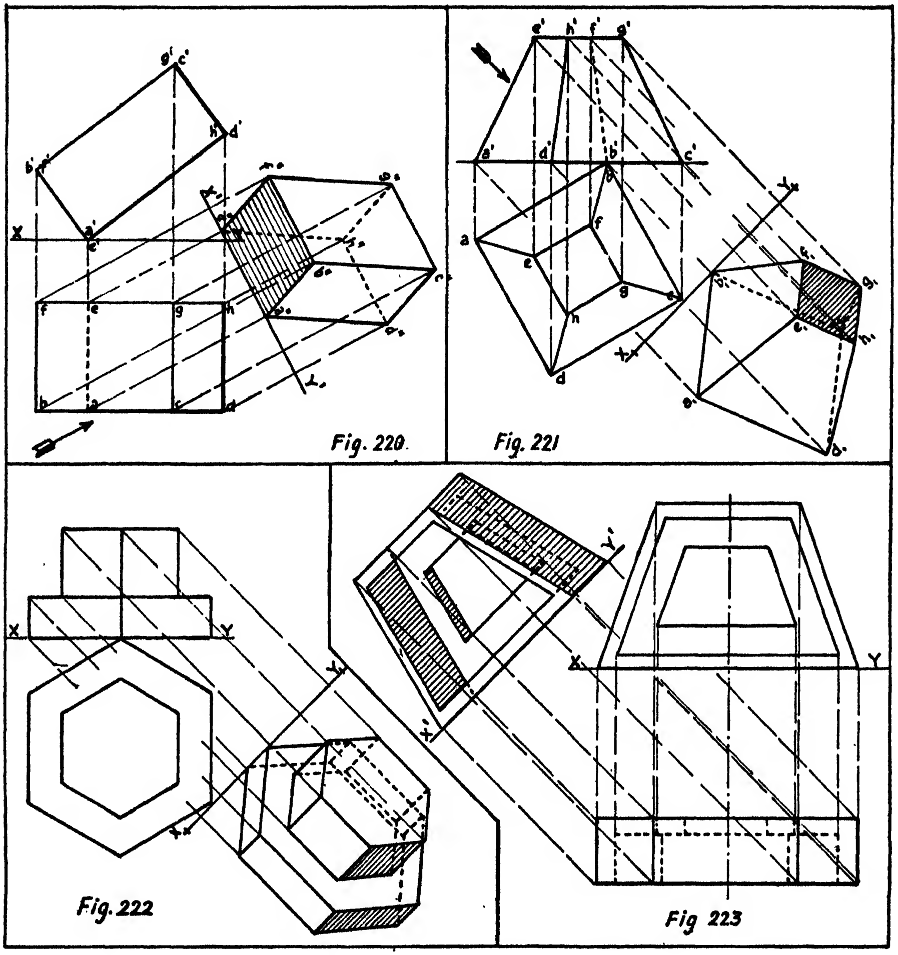


Fig. 220. Fig. 221

Fig. 222 Fig. 223

time, confusion is caused and errors introduced through transferring distances on the wrong projectors.

Fig. 222 represents the plan and elevation of a hexagonal flanged nut and the projection of a new plan on a plane represented by X_1Y_1.

Fig. 223 gives the plan and elevation of a cast-iron bracket and illustrates the projection of a new elevation on a plane X^1Y^1.

CHAPTER XV

SECTIONAL VIEWS

In preparing working drawings it is frequently necessary, in order that hidden details and internal construction may be made clear, to draw views representing projections of the object when it is assumed to have been cut through by a plane and the portions between the section plane and the observer removed. The cut surface is generally distinguished from the uncut portions by suitable cross hatching, as explained in Chapter III.

To illustrate this figs. 224, 225, and 226 show projections of a cube cut in each case by a plane represented by the line SP. The section plane is horizontal in fig. 224, vertical in fig. 225, and inclined in fig. 226.

Figs. 227 to 232 show projections of a square pyramid cut by section planes SP in each case. The method of projecting the outline of the cut surface will be explained by reference to fig. 231 as a typical example.

Where the line SP in elevation, representing the section plane, cuts an edge of the object as at a', b', c', and d', these points of intersection are projected on to the plans of the edges and joined in sequence to obtain the plan of the cut surface.

It is to be noted in figs. 224, 227, 228, and 232 that the projection of the cut surface displays the true shape of the section because the section plane is perpendicular to the line of sight. In the remaining figures the true shape is not displayed, and to obtain it in these cases an auxiliary view would require to be drawn looking at right angles to SP as indicated by the arrows.

EXERCISES

1. The cube represented in fig. 233 is of $2\frac{1}{4}$ in. edge and is cut by a plane which passes through the corners A, B, and C. Draw plan and elevation of the larger portion and project a view showing the true shape of the cut surface.

2. Fig. 234 is a cross-section of a ring to be turned in the lathe, the outside diameter D being 6 in. Draw a plan and elevation of the ring $\frac{1}{2}$ full size.

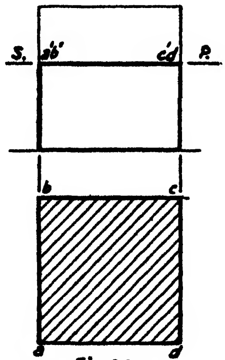


Fig. 224

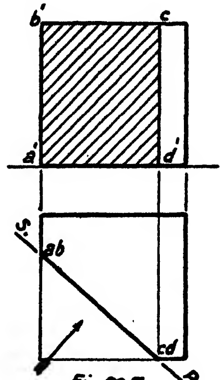


Fig. 225

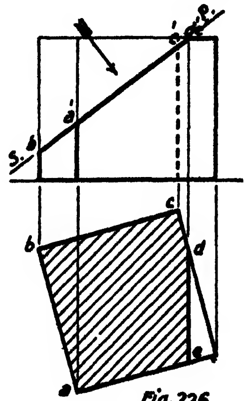


Fig. 226

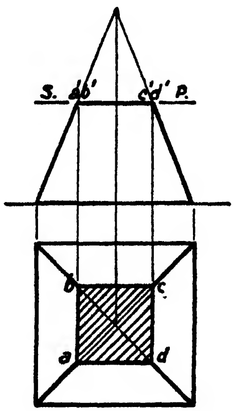


Fig. 227

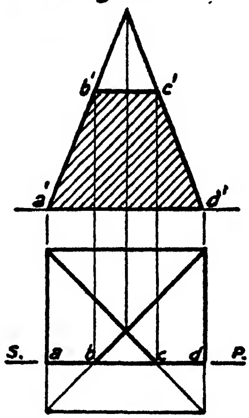


Fig. 228

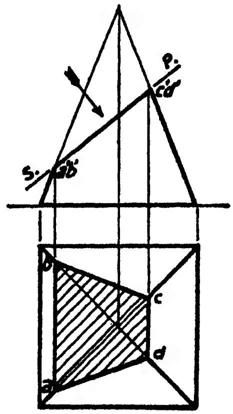


Fig. 229

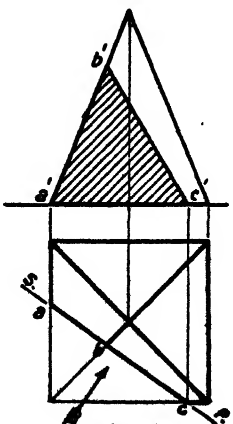


Fig. 230

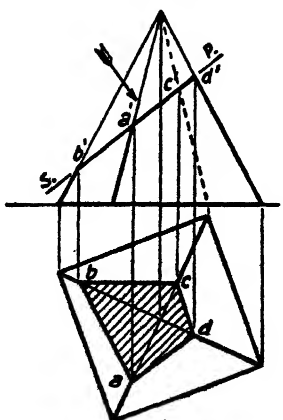


Fig. 231

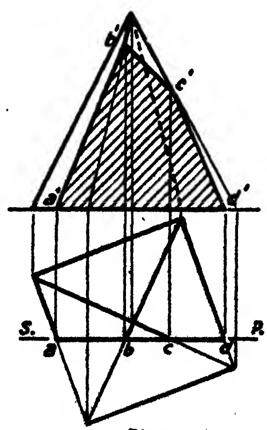


Fig 232

3. Projections of a truncated triangular pyramid are given in fig. 235. It is cut by a plane which passes through the corners A, B, and C. (*a*) Draw the views given, showing the cut surface on each. (*b*) Project a side elevation of the lower portion looking in the direction of the arrow. (*c*) Obtain the true shape of the section.

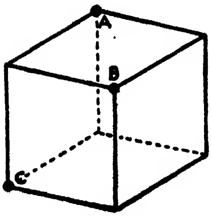


Fig. 233

4. Projections of a truncated square pyramid are given in fig. 236. It is cut by a plane which passes through the edges AB and CD. Draw the views, showing the cut surface on each, project a side elevation looking in the direction of the arrow, and obtain the true shape of the section.

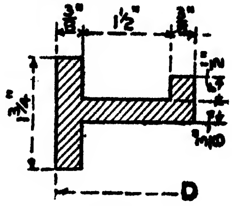


Fig. 234

5. Fig. 237 gives a section of a moulded dado rail which is mitred at the angle of a wall as shown in the small plan attached. Draw the true shape of the mitre cut ½ full size.

6. The moulding in fig. 70 is mitred (*a*) in the direction of its width, i.e. as applied as a moulding to a rectangular door panel; (*b*) in the direction of its thickness, i.e. as applied as a picture rail. Draw full size the true shape of the mitre cut in each case.

7. Fig. 238 gives a plan and uncompleted elevation of a cast-iron bracket. Draw these to a scale of 1½ in. to 1 ft. Complete the elevation, and project a sectional end elevation looking in the direction of the arrow when the casting is cut by a plane represented by AA.

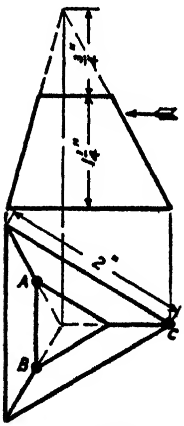


Fig. 235

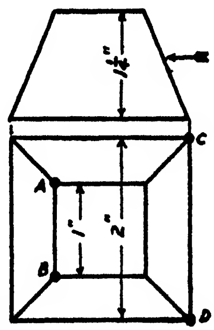


Fig. 236

Conic Sections.—A cone may be defined as the surface swept out or generated by a line inclined at a constant angle to, and intersecting a fixed line about which it rotates. The fixed line is the **axis**, the rotating line is called a **generator**, its angle of inclination to the axis is the **semi-vertical angle** of the cone, and the point of intersection of the lines the **vertex** of the cone.

The sections of a cone made by a plane produce a number of curves of particular interest and importance which from their association are known

as the **conic sections.** According to the position of the section plane, these sections take the following forms:—

A **triangle,** when the plane passes through the vertex (fig. 239).

A **circle,** when the plane is perpendicular to the axis (fig. 240).

An **ellipse,** when the plane is inclined to the axis at an angle greater than the semi-vertical angle but less than a right angle (fig. 241).

A **parabola,** when the plane is inclined to the axis at an angle equal to the semi-vertical angle, i.e. when it is parallel with a generator (fig. 242).

A **hyperbola,** when the plane is inclined to the axis at an angle less than the semi-vertical angle (fig. 243). *Note.*—In the figure the plane has been selected parallel with the axis.

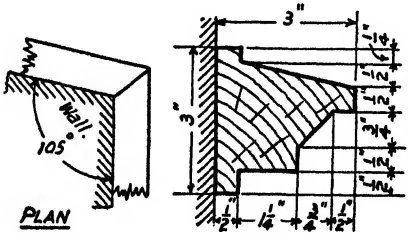


Fig. 237

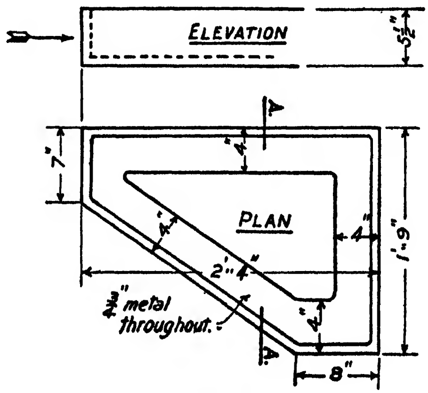


Fig. 238

In fig. 239 the intersection of the plane SP with the base in elevation is projected on to the plan of the base at *a* and *b*. Lines joining these points to each other and to the plan of the vertex give the projection of the section. The true shape would be found by projecting a view looking at right angles to S.P. in the direction of the arrow.

In fig. 240 the intersections of SP with the extreme generators in elevation are projected to the plan at *a* and *b*. The circle described on this as diameter gives the plan and also the true shape of the section.

In figs. 241 to 243 the method employed to obtain the sections and true shapes is the same in each case, and is here described for fig. 241.

Draw any line perpendicular to the axis, such as *a'b'* in elevation, to represent the elevation of a circle on the surface of the cone. Project the points *a'b'* to the plan at *ab*, and draw the plan of the circle with *ab* as diameter. From the point where SP cuts *a'b'* draw a projector to intersect the circle on plan at *cd*. These will be two points on the plan of the section.

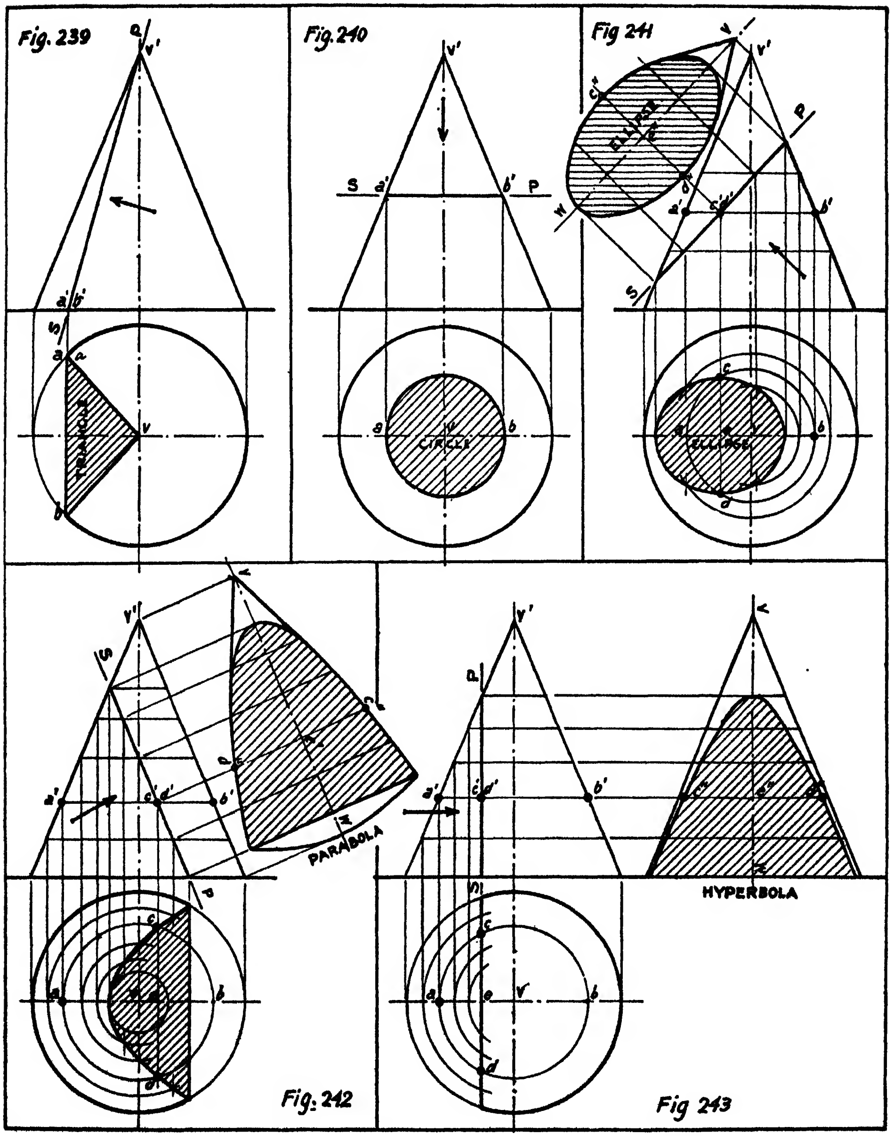


Fig. 239
Fig. 240
Fig 241
CIRCLE
ELLIPSE
PARABOLA
HYPERBOLA
Fig. 242
Fig 243

Repeat the process for a number of similar circles, and join the points on plan by a fair curve to obtain the plan of the section.

To obtain the true shape a view at right angles to SP in the direction of the arrow must be projected thus:—

Draw a line VW parallel to SP at a convenient distance to ensure that the views will not overlap. From the point *c′d′* on SP draw a projector and set out thereon on each side of VW the half widths *ce*, *ed*, from the plan to obtain the points *c″d″* Repeat this from the other intersections and join the points by a fair curve.

Apart from their derivation as sections of a cone as described above, the need for setting out ellipses and parabolas from given data is of such frequent occurrence that the following notes and constructions are given.

The Ellipse

This curve is the path traced out by a point which moves so that the sum of its distances from two fixed points called the **foci** is constant.

For the following definitions refer to fig. 244:

Centre.—A point (O) midway between the **foci** (F and F_1).

Diameter.—Any line through the centre terminated at each end by the curve (ST).

Major Axis.—The diameter passing through the foci. It is the longest diameter of the curve (AB).

Minor Axis.—The diameter which bisects the major axis perpendicularly. It is the shortest diameter of the curve (CD) and is bisected perpendicularly by the major axis.

Major and Minor Auxiliary Circles.—The two circles having the major and minor axes respectively for diameters.

The sum of the distances EF, EF_1 of any point, such as E on the circumference, from the foci F and F_1 is constant, and equal to AB.

Given the Major and Minor Axes, to Find the Position of the Foci.—With C (an extremity of the minor axis) as centre and radius equal to AO (half the major axis), describe an arc of a circle to cut the major axis in F and F_1 the required foci.

To Describe an Ellipse, given the Major and Minor Axes.—*Method 1.*—Locate the foci and insert a pin at each, and another at C, one end of the minor axis. Carefully tie a fine thread round these in the form of a triangle. Remove the pin at C, and place the pencil point in

the loop of the thread. Move the pencil round, keeping it in the loop, and, maintaining an even tension in the thread, mark out the required ellipse on the paper.

While this method illustrates practically the truth of the above definition, unless the ellipse is large it is not easy to maintain accuracy by its use, and the following constructions are preferable:

Method 2.—Draw the auxiliary circles, and any common radius, such as OQR. Through Q and R draw perpendiculars to the axes, as indicated, to intersect at P, which will be a point on the curve. Repeat this for a number of radii, and draw the curve by freehand to pass through the points thus obtained (fig. 244).

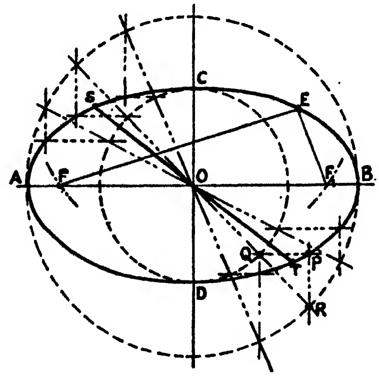


Fig 244.

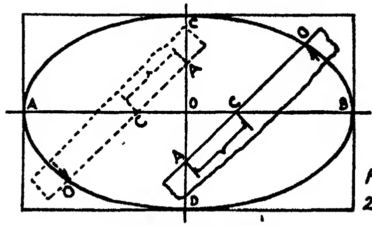


Fig 245

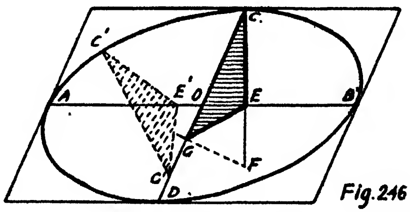


Fig. 246

Method 3 (Trammel method).—On the edge of a piece of paper (or on a line ruled on tracing paper) mark off OA (half the major axis) and OC (half the minor axis). This forms what is known as a "trammel". If the trammel is now placed in successive positions, so that A is always on the minor axis while C is always on the major axis, and the positions of the point O marked on the paper, these points will all lie on the curve, which may be drawn by freehand to pass through the points so found (fig. 245).

The foregoing methods can be applied to inscribe an ellipse in a rectangle to touch the mid-points of the sides.

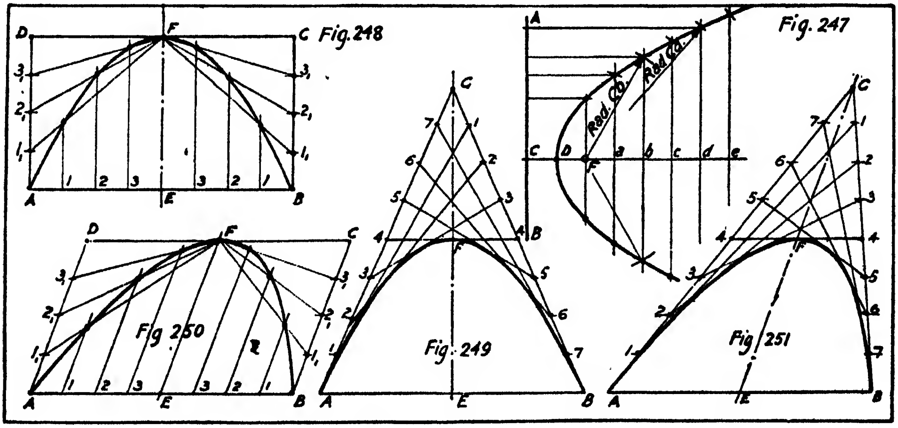
To Inscribe an Ellipse in a Parallelogram to touch the Mid-points of the Sides (fig. 246).—Bisect the sides and join by lines AB and CD intersecting at O. From C draw CF perpendicular to AB intersecting

AB at E, and make CF equal to AO. Draw FG perpendicular to CD and intersecting it at G. Join G to E. The triangle CEG will form a **triangular trammel.** Trace this on tracing paper and place it in successive positions, so that E is always on AB while G is always on CD. Prick through the positions of the point C and join the points to obtain the required ellipse.

The Parabola

This curve is the path traced out by a point which moves so that its distances from a fixed point and a fixed straight line are equal.

The fixed point is the **focus**, the fixed line the **directrix**, the line through the focus perpendicular to the directrix the **axis**, and the point where the curve intersects the axis the **vertex**.



To Set out a Parabolic Curve, given the Focus (F) and Directrix (AB) (fig. 247).—Through F draw a line perpendicular to AB meeting it at C; this line will be the axis. Bisect CF to obtain D, a point on the curve; this will be the vertex. On the axis set out any points such as *a*, *b*, *c*, &c., and draw through these lines perpendicular to the axis. With the focus F as centre and radius CF, describe arcs cutting the perpendicular through F in two points on the curve. Proceed, with F as centre and radii C*a*, C*b*, &c., to describe arcs cutting the perpendiculars through *a*, *b*, &c. The curve drawn through the points of intersection will be the required parabola.

To Draw a Parabolic Curve on a given Base (AB) and of given Height (EF) (fig. 248).—Complete the rectangle ABCD. Divide AE and

BE into any number of equal parts as at 1, 2, 3, and draw perpendiculars to AB. Divide AD and BC into the same number of parts as at 1_1, 2_1, 3_1, and join to F.

The intersections of $F1_1$, $F2_1$, &c., with the perpendiculars through 1, 2, &c. respectively, give points on the curve.

Alternatively (fig. 249): Produce EF to G, making FG equal to EF. Divide AG and BG each into an even number of equal parts as at 1, 2, 3, &c., numbering the points of division as shown in the figure.

Join 1 to 1, 2 to 2, and so on. Each of these lines will be a tangent to the parabola, and a fair curve drawn to touch all of them will be the required parabola.

To Draw a Parabolic Curve on a given Base (AB) and to touch a Parallel Line (CD) at a given Point (F).—Similar constructions to those described above are to be followed except that EF is not perpendicular to AB, and the methods are made clear in figs. 250 and 251.

EXERCISES

8. Inscribe an ellipse in a rectangle 3·2 in. × 2·1 in.

9. Draw a circle 4 in. diameter; draw a diameter and any number of chords perpendicular thereto. Bisect all the semichords and join the points of division. Verify the fact that the curve so drawn is an ellipse. Trisect the semichords and join the points of division. Note that these curves also are ellipses.

10. Draw two lines, 4 in. and 2½ in. long, bisecting each other at an angle of 70°. Construct a triangular trammel, and by its use draw the ellipse which has these lines for its axes.

11. Draw an isometric projection of a square of 3 in. side. Considering this as a parallelogram, set out by the use of a triangular trammel the ellipse representing the isometric projection of a circle inscribed in the square.

12. Describe a parabolic segment on a base 3 in. long, the height to the vertex being 2½ in.

13. A jet of water issues horizontally from a nozzle 20 ft. above ground level and strikes the ground at a point 18 ft. from the point on the ground vertically below the nozzle. Draw the path of the jet ⅛ in. to 1 ft.

Note.—Neglecting air resistance, a jet of water follows a parabolic path.

Curves derived from Intersections.—In machine details examples frequently arise of curves formed by the intersection of circular parts with plane surfaces, and these can be treated as examples of plane sections.

For example, figs. 252 and 253 represent the two ends of a connecting rod. This is turned in the lathe, the lathe centres being on the axis of the rod. The ends are then planed to the widths shown in the sectional views. The method of obtaining the outline of the curves in which the sides of the foot or the forked end intersect the turned portions is indicated in the figure.

In fig. 252 a section of the neck perpendicular to the axis is represented on the elevation by the line LM and on the sectional elevation by the

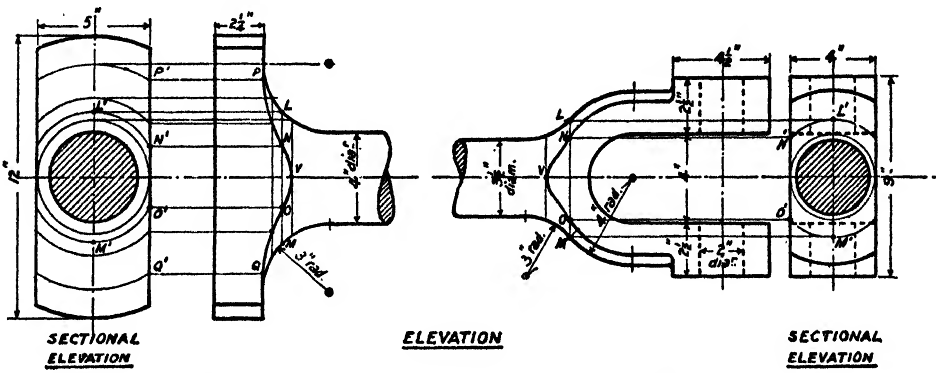


Fig. 252 Fig. 253

circular arcs L′M′. Where these arcs intersect the width of the foot at N′O′, draw projectors to the elevation meeting LM in the points N and O to obtain 2 points on the required curve. Take other similar sections to obtain further points. The vertex of the curve V is found by taking a section the diameter of which is equal to the width of the foot. The commencing points PQ of the curve are found by taking a section at the level of the top of the foot.

To obtain the curve in fig. 253 the procedure is the same, and the same lettering is used so that the above description shall apply.

These examples should be drawn ½ full size to the dimensions given.

EXERCISES

14. Fig. 254 represents plan and incomplete elevation of a column with a 3 in. square base which is turned to the profile shown in elevation. Draw full size, complete the elevation by adding the curves of intersection, and project an auxiliary view which shows the true shape of one face of the base.

15. Fig. 255 represents plan and incomplete elevation of the foot of a column turned out of 4 in. × 2½ in. material to the profile shown in elevation. Draw full size, complete the elevation by adding the curve of intersection, and project a side elevation showing the curve of intersection on the narrow face.

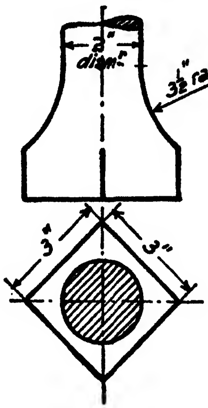


Fig. 254

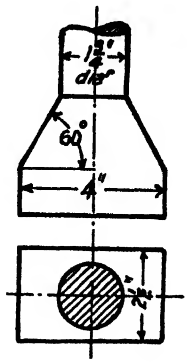


Fig. 255

CHAPTER XVI

INTERSECTION OR INTERPENETRATION OF SOLIDS

The determination of the intersection made when two solids either join or penetrate each other is of frequent occurrence in practical drawing. In buildings, the intersections of turrets, chimneys, &c., with the roof slopes, and of roof slopes with each other, in sheet-metal work, the intersection of ducts and pipes of various shapes, or in machine details, the intersection of curved parts with each other have to be set out and form an important branch of practical drawing.

A few of the simpler and more common examples are dealt with here.

Interpenetration of Plane-faced Solids.—Examples of the intersection of **two square prisms, the axes of which intersect each other,** are shown in fig. 256, with the method of determining the outline. On the left of the figure the solids are shown perpendicular to each other, and on the right are inclined.

The intersection of the edges of the two solids and of the edges of one with the faces of the other must be obtained and joined in sequence to obtain the complete outline of the intersection.

The edges which lie in the plane containing the axes display their intersection directly at *a*, *c* and *e*, *g* in plan and elevation.

The remaining edges of the smaller prism intersect the faces of the larger one at *b*, *d* and *f*, *h* on plan. Project these points to the elevation and join as shown to obtain the completed intersections *a*, *b*, *c*, *d* or *e*, *f*, *g*, *h*.

Examples of **two intersecting prisms, the axes of which do not intersect,** are shown in fig. 257.

The intersection of the edges of the smaller prism with the faces of the larger are obtained directly from the plan at *a*, *c*, *e*, *f* and *g*, *i*, *k*, *l*. Project these points to the elevations of the corresponding edges. The intersections of the edges of the larger prism with the faces of the smaller are seen in plan at *b*, *d* and *h*, *j*, but to obtain their positions in elevation an auxiliary side elevation—shown by thin lines—is necessary from which the points b'', d'' and h'', j'' are obtained. Project these to the elevation

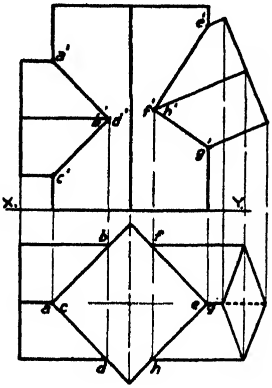


Fig 256

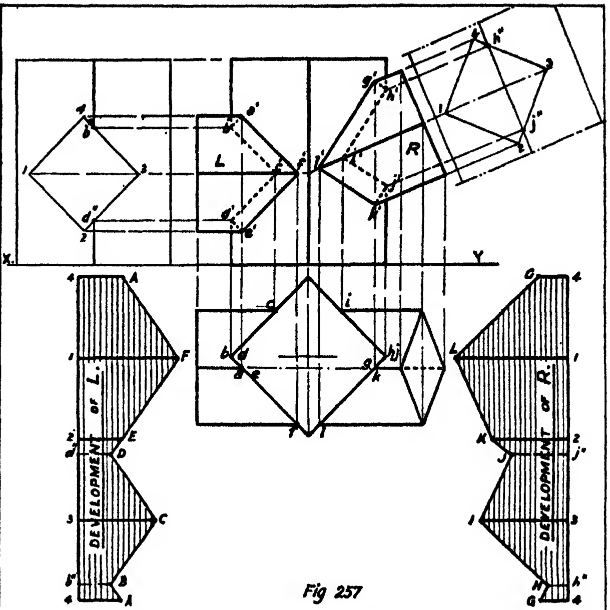


Fig 257

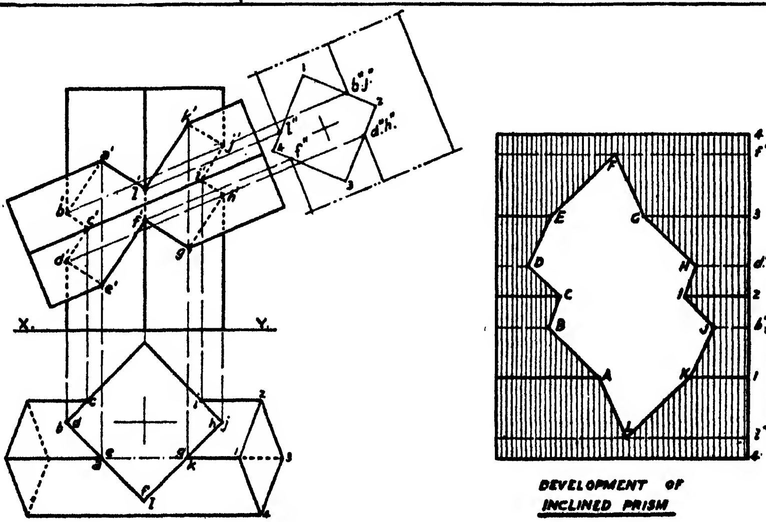


Fig 258

and join in sequence to obtain the completed outlines *a, b, c, d, e, f* and *g, h, i, j, k, l*.

The case of the inclined intersection of **two prisms neither of which completely penetrates the other** is shown in fig. 258.

The intersections of the edges of the smaller prism with the faces of the larger are shown in plan at *a, c, e* and *g, i, k*. Project these to the corresponding edges in elevation. Draw an auxiliary side elevation as shown by thin lines to locate the points b'', d'', h'', j'', l'', f'', where the edges of the larger prism intersect the faces of the smaller. Project these points back to the elevation and join in sequence.

An example of a **square prism penetrated by a pyramid, the axes intersecting,** is shown in fig. 259. The edges lying in the plane containing the axes display their intersections directly at *a, c, e, g* and at the corresponding points in elevation. An auxiliary side elevation is required to locate the points b'', d'', f'', h'' in which the remaining edges of the pyramid intersect the faces of the prism. Project these to the elevation and join in sequence to complete the outlines in elevation. Transfer the half-widths of $b''d''$ and $f''h''$ to the plan on each side of the centre line to obtain *b, d, f* and *h* and complete the outlines in plan.

The case of a similar **square prism penetrated by a pyramid, the axes not intersecting,** is shown in fig. 260. A side elevation as shown is necessary. From this the points a'', g'', c'', e'', k'' and i'' are located, and projected to the corresponding edges in elevation and thence to the plan.

To locate the remaining points on the intersection imagine a horizontal section of the two solids to be made. The section of the pyramid will be a square and of the prism a rectangle. The points where square and rectangle intersect will obviously lie on the intersection of the solids.

The method is illustrated in the figure for one such horizontal section passing through the axis of the prism. The section of the pyramid is the square *mnop* shown by thin lines in plan, and of the prism the rectangle *qrst*. The line *st* does not intersect the square, showing that this edge does not penetrate the pyramid, but *qr* intersects the square at *d* and *j*, which are points on the plan of the required intersection.

Similar horizontal sections must be taken at the top and bottom edges of the prism to obtain the remaining points.

For clearness a section through the top edge is drawn, separated from the plan to indicate how the points *b* and *l* are obtained.

Interpenetration of Cylinders and Cones.—If two cylinders, or a cylinder and a cone, or two cones, intersect each other at any angle, and the curved surfaces of both solids **enclose the same sphere,** the outline of the intersection in each case will be **an ellipse.**

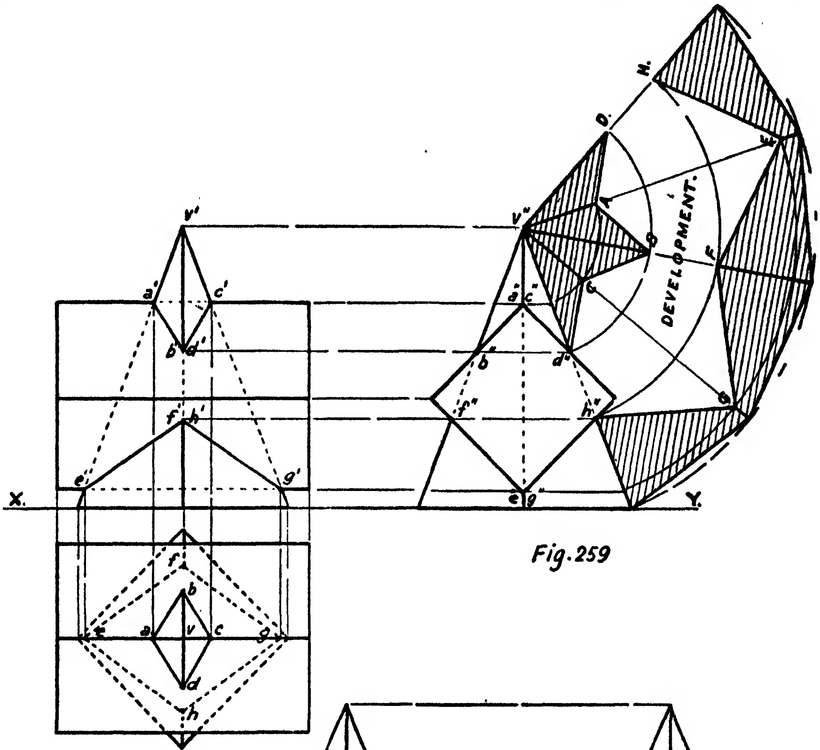


Fig. 259

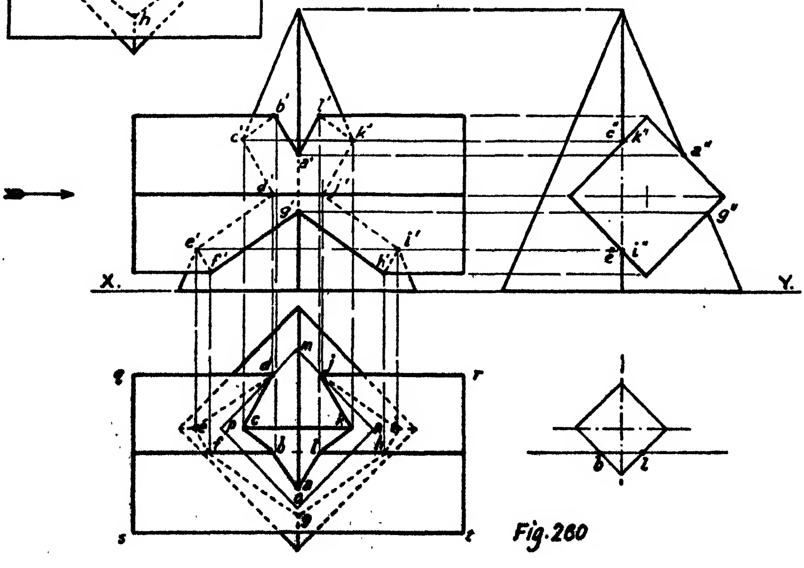
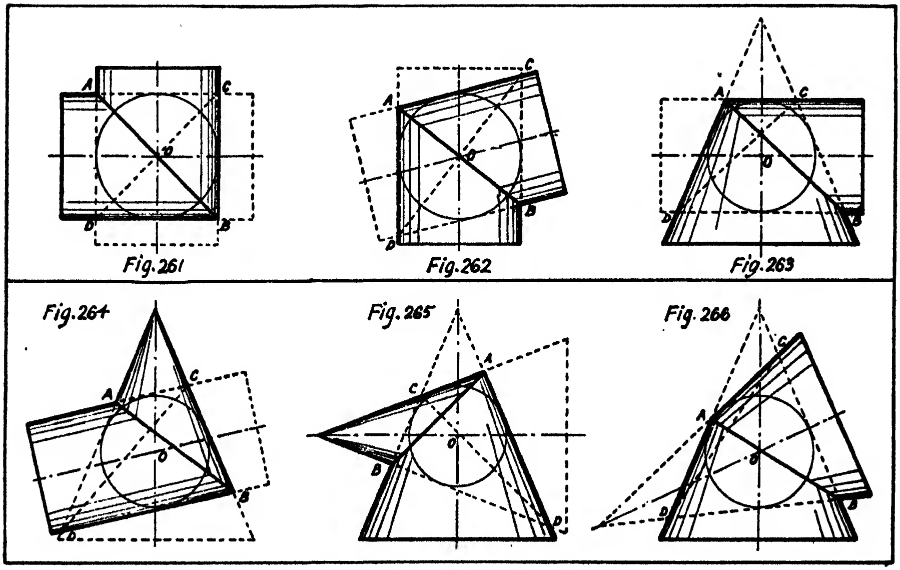


Fig. 260

Such types of intersection are illustrated in figs. 261 to 266, and the enclosed sphere indicated in each case. The condition that the solids enclose the same sphere involves that the **axes intersect** at the centre of the sphere as at O. In the case of two cylinders it also involves that they are equal in diameter. In a view looking at right angles to the plane containing the axes, the intersection will be an edgewise view of the ellipse and will thus appear as a straight line as at AB.

If the solids interpenetrate completely as shown by dotted lines, the complete intersection will include a second ellipse as at CD.



The intersection of **two unequal cylinders, the axes of which intersect** (fig. 267). On the left of this figure the solids are at right angles, and on the right are inclined.

Draw auxiliary circles representing end views of the smaller cylinder in plan and elevation and divide the circumference into a number (say 12) of equal parts. In the beginning stages students should number these points of division to correspond in plan and elevation as indicated. Through the points of division draw lines parallel to the axis of the cylinder in both views. (These will represent generators of the cylinder.) Consider the lines through points 4 and 2. The intersection of the line on plan through these points with the circumference of the larger cylinder at P is on the required curve. Project from P to meet the lines through 4 and 2 in

elevation. Repeat the process for the remaining generators and draw fair curves through the points in elevation.

Interpenetration of a Cylinder by a Cone, the Axes Intersecting (fig. 268).—An auxiliary side elevation, shown by thin lines, is necessary. On this side elevation divide the circumference of the cylinder into a number (say 12) of equal parts. Project the points of division to the elevation to represent generators of the cylinder. Draw plans of these

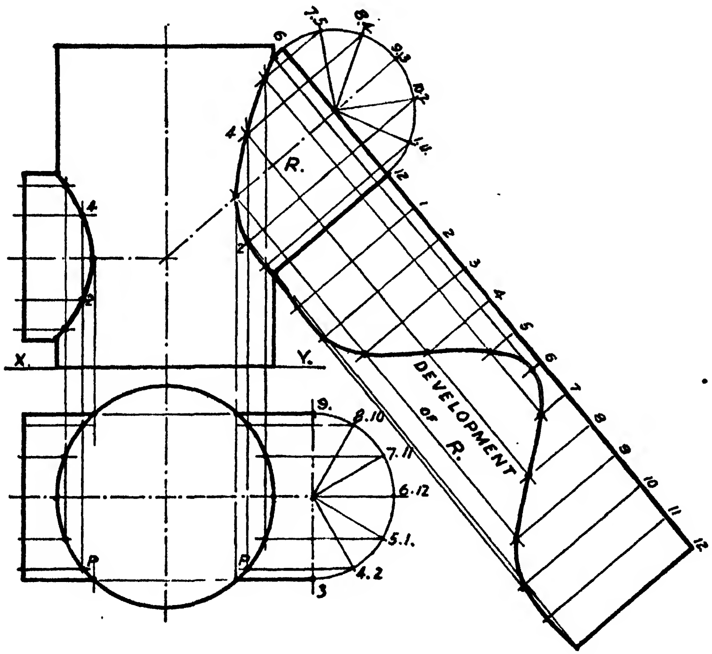


Fig. 267

generators by the aid of the auxiliary circle shown in plan, and number the lines to correspond in both views.

Consider a horizontal section through the points 4, 8. This will cut the cone in a circle of radius *ab* obtained from the side elevation. With radius *ab* and centre *v* on plan, draw a circle to cut the lines through 4 and 8 in *c*, *d*, *e*, *f*. These will be on the plan of the intersection curve. Project these to the elevation at c', d', e', f'.

Repeat the process for horizontal sections through 12, 1.11, 5.7, and 6, and also at the points k'', l'', m'', n'', on the side elevation and join the points on plan and elevation by fair curves.

Intersection of two Unequal Cylinders, the Axes of which do not Intersect (fig. 269).—The explanation and methods given in the foregoing for fig. 267 apply to this case, and taken in conjunction with the figure make the construction clear.

Intersection of Cylinder and Cone, the Axes of which do not Intersect (fig. 270).—The method of horizontal sections described in the

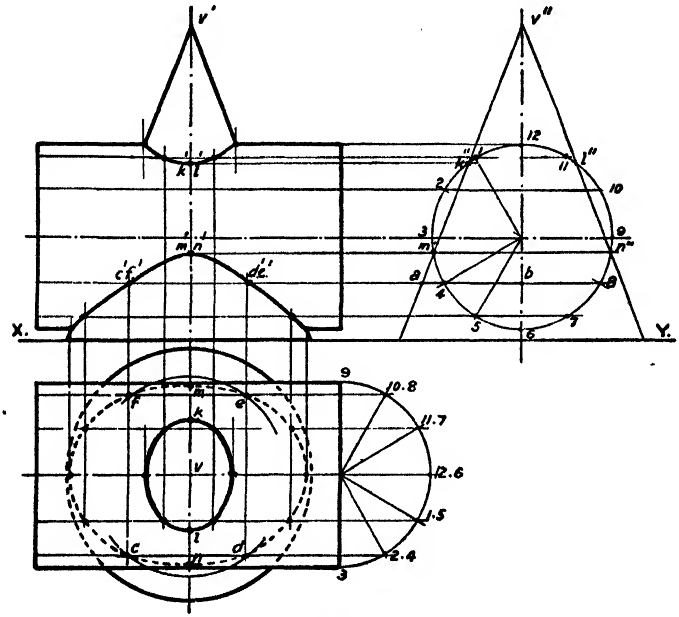


Fig. 268

foregoing for fig. 268 apply to this case. The construction is made clear in the figure for a section through the points 8.10 to obtain the points *c, d, e, f* in plan on the intersection curve.

EXERCISES

1. Fig. 271 shows an incomplete elevation of a vertical timber fence post 4 in. square and an inclined strut 3 in. square. Complete the elevation by setting out the intersection. Draw a side elevation of the post looking in the direction of the arrow, assuming the strut removed and showing the notch or " birdsmouth ". Scale ¼ full size.

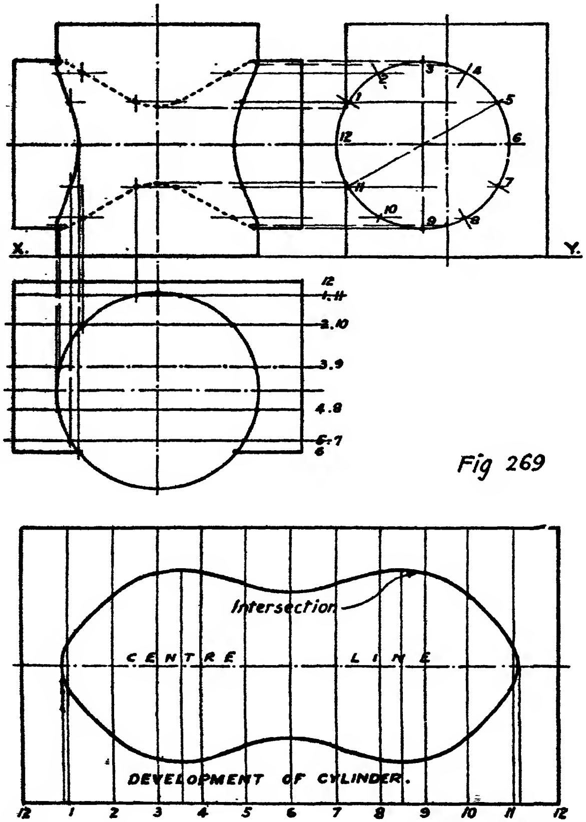


Fig 269

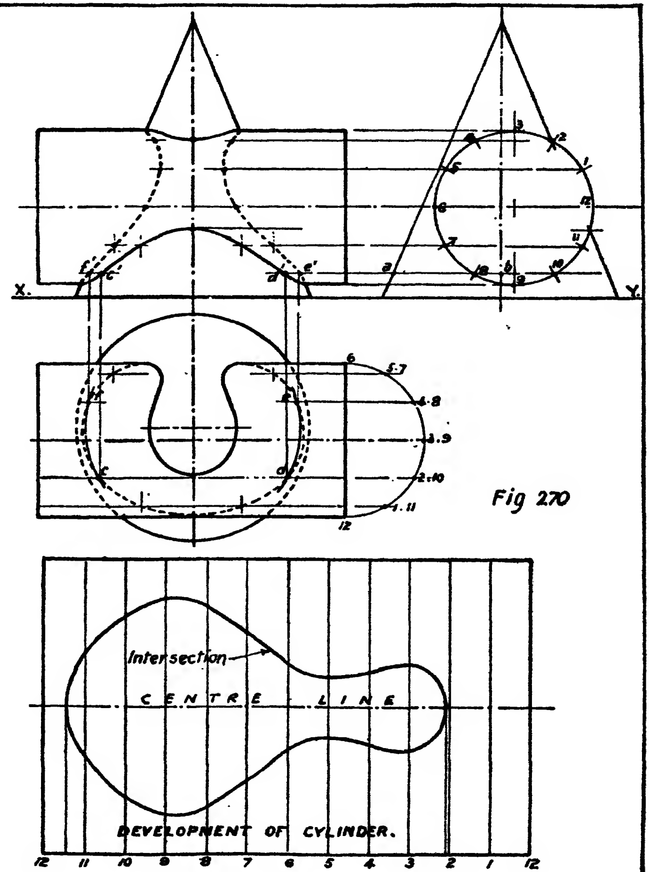


Fig 270

2. Fig. 272 shows a plan of a vertical post 4 in. square and a strut 4 in. diameter which meets it at an angle of 30° with the vertical. Draw an elevation on XY showing the curve of intersection. Scale $\frac{1}{8}$ full size.

3. Fig. 273 shows elevation and section of a moulding cut to fit into a circular recess. Project a plan showing the outline of the intersection. Scale $\frac{1}{2}$ full size.

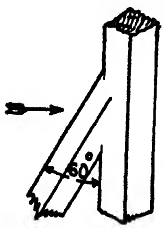


Fig. 271

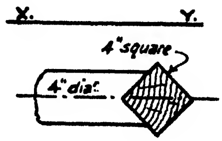


Fig. 272

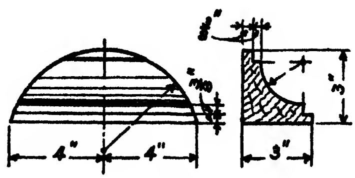


Fig. 273

4. Fig. 274 shows in plan a junction in a rainwater pipe 4 in. outside diameter with a $3\frac{1}{2}$ in. diameter curved branch. Draw $\frac{1}{2}$ full size, adding the intersection curve, and draw an elevation.

5. Fig. 275 shows a 6 in. diameter sphere pierced with a hole $2\frac{1}{2}$ in. diameter, the centre of the hole being $1\frac{3}{8}$ in. from the centre of the sphere. Draw an elevation showing the outline of the hole. Project a new elevation looking in the direction of the arrow. Scale $\frac{1}{2}$ full size.

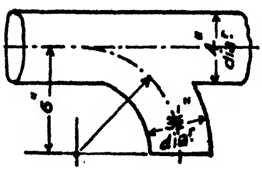


Fig. 274

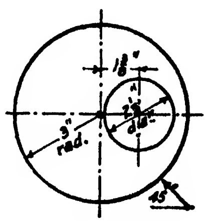


Fig. 275

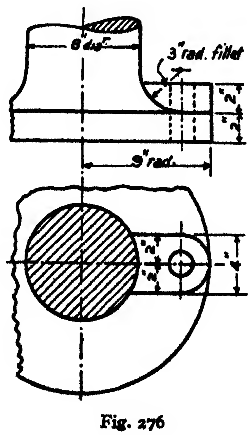


Fig. 276

6. Fig. 276 gives a plan and incomplete elevation of a portion of a cast-iron column base, with a boss for the reception of a holding-down bolt. Draw these $\frac{1}{4}$ full size, and complete the elevation by showing the intersection of the boss with the curved fillet of the base.

CHAPTER XVII

DEVELOPMENTS OF SURFACES

The general idea of the development of the surface of a solid has been touched upon in Chapter X. Further examples of this important branch of practical drawing will now be considered.

Development of Plane-faced Solids

In fig. 257 developments are given of the right- and left-hand portions of the square prisms which intersect the vertical one. These are such as a sheet-metal worker would have to make if for instance the prisms were metal ducts.

Dealing with development R. On the line 4.4 set out lengths 4.1, 1.2, &c., equal to the side of the prism end. The distances from 3 to h'' and j'' are obtained and transferred from the auxiliary side elevation. On perpendiculars through 1, 2, 3, and 4, h'' and j'', transfer from the elevation the true distances from l, k, j, &c., to the end of the prism.

Straight lines joining G, H, I, J, K, and L complete the development of the intersecting end.

Similar methods are used to obtain the development in fig. 258 of the inclined smaller prism.

In fig. 259 a development of the pyramid is given, showing the gap left by the penetrating prism.

With centre V$''$ and radius equal to the length of the edge of the pyramid, describe an arc and on this step out the length of the base four times and draw radii to V$''$. From the points on the side elevation where a horizontal line through $a''c''$ meets the edge of the pyramid, draw an arc from V$''$ as centre to cut the corresponding radii at C and A. Repeat for the remaining points and join by straight lines to complete the outline of the gap.

Development of Cylindrical and Conical Surfaces

The development of a cylinder is a **rectangle** having one side equal to the circumference of the base and the other to the length of the cylinder.

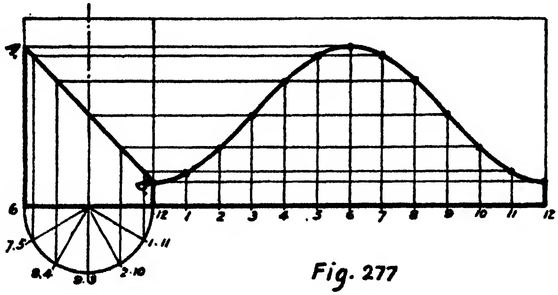


Fig. 277

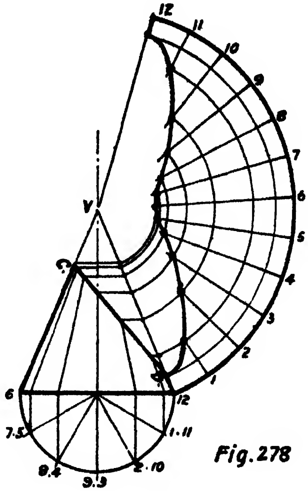


Fig. 278

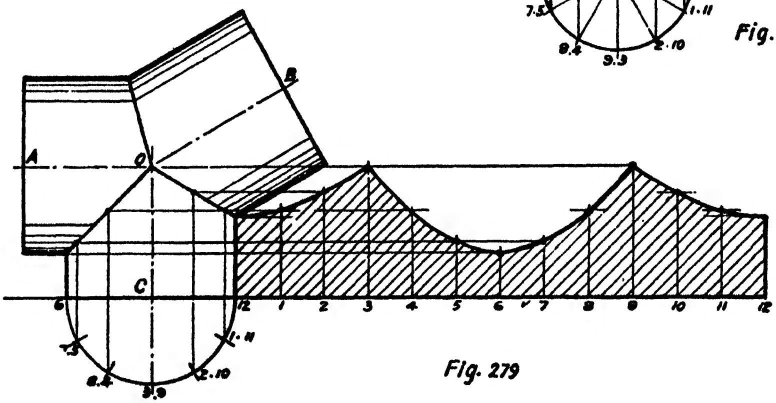


Fig. 279

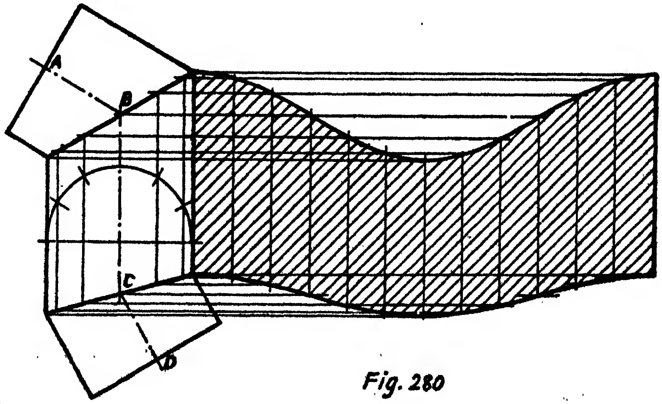


Fig. 280

Fig. 277 shows the method of developing the cylinder and also of the portion cut off by an inclined plane such as AB.

Divide the circumference into a number of equal parts, step these out along a line such as 12.12, and draw perpendiculars through the points of division. Project generators to the elevation, and from the points where they cut AB draw lines parallel to 12.12 to meet the developed positions of the corresponding generators and draw the curve.

Similar procedure would serve to develop any of the cylindrical portions in figs. 261 to 264.

The development of a cone is a **sector**, of radius equal to the slant side of the cone, and the length of the arc is equal to the circumference of the base.

Fig. 278 shows the development of the cone and also of the portion cut off by an inclined plane such as CD.

With centre V and radius V.12, describe an arc and step out along this the lengths into which the base is divided. Draw radii through V to obtain the development of the cone.

Project generators to the elevation, and where these cut CD draw parallels to the base. From the points where these parallels meet the slant side describe arcs with centre V to meet the corresponding developed generators and draw the curve.

The development of any of the conical portions in figs. 263 to 266 would be done in a similar way.

Example.—Fig. 279 shows 3 equal cylindrical pipes the axes of which intersect and the method of developing one portion. Draw this, making the cylinders 2 ft. diameter and the angles AOC and BOC 90° and 125° respectively. Scale 1 in. to 1 ft.

Example.—Fig. 280 represents a double elbow in a cylindrical duct and the development of the junction piece. Draw this, making the duct 1 ft. 6 in. diameter, the angles ABC and BCD 110° and 145° respectively, and the length BC 2 ft. 3 in. Scale 1½ in. to 1 ft.

Example.—Fig. 281 shows two cones intersecting at right angles and enclosing the same sphere, and the development of the lower portion of the vertical cone.

To obtain the development of the point P, join V to P and produce to cut the base in Q. Project Q to the circumference in plan at R. Transfer the distances 3.R and 9.R to the development at R_1 and R_{11} to obtain the radii on which the development of P will be situated. The remainder of the construction follows the methods already described. Draw this, making the enclosed sphere 2¼ in. diameter and the distances VO and WO 2¾ in. and 3¼ in. respectively.

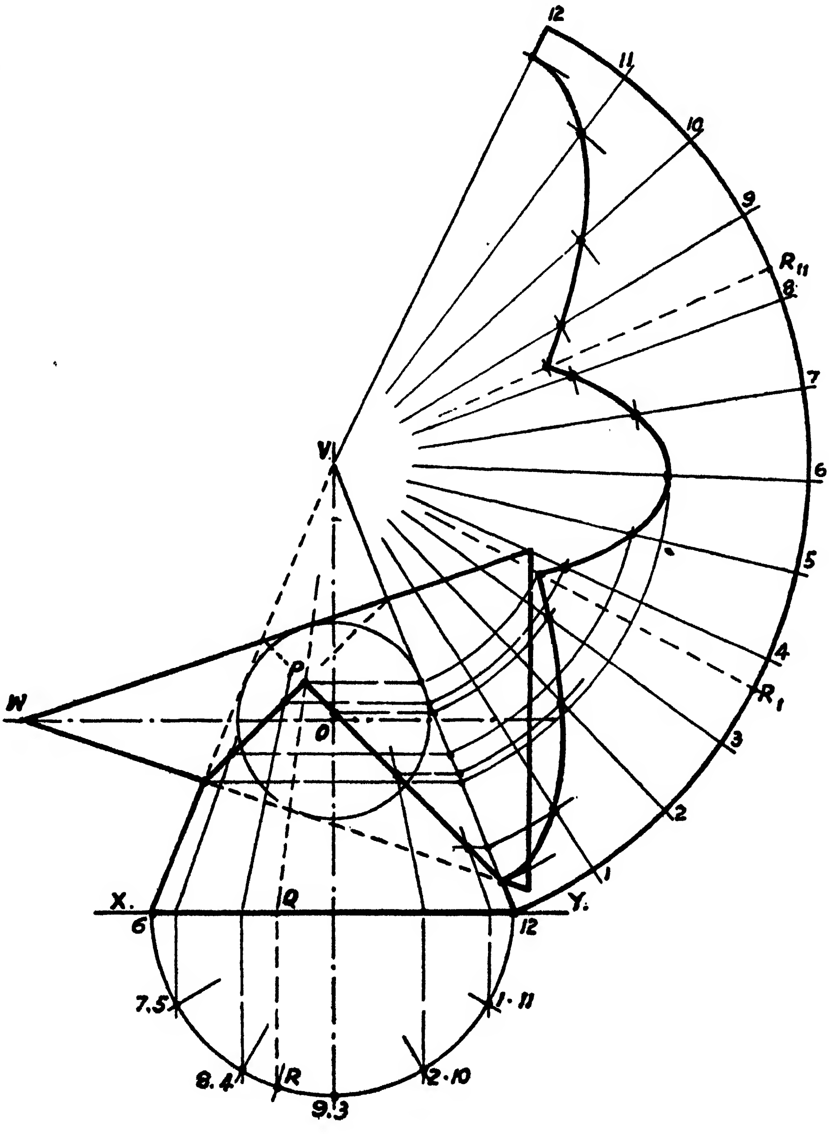


Fig. 281

Figs. 269 and 270 show the developments of the horizontal cylinder in each case with the development of the curve of intersection. The construction follows the methods described above, and the distances to the curve from the centre line can be obtained directly from the elevation in each case.

EXERCISES

1. The bottom portion of a steam dome consists of a cylindrical plate 18 in. diameter. The boiler is 5 ft. diameter, and the axis of the steam dome is 9 in. from the axis of the boiler (fig. 282). Develop the plate of the dome, neglecting laps or flanges. Scale $1\frac{1}{2}$ in. = 1 ft.

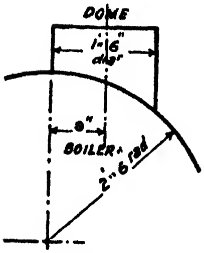


Fig. 282

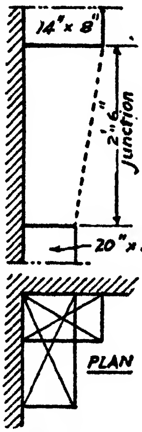


Fig. 283

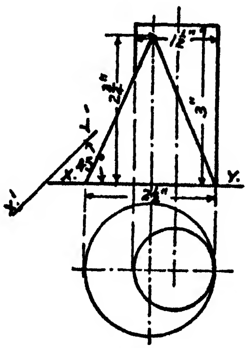


Fig. 284

2. An air trunk at the vertical angle of a room is shown in plan (fig. 283). The lower portion is 20 in. × 9 in., and a junction piece 2 ft. 6 in. long is required to connect this to the upper portion 14 in. × 8 in. Neglecting laps and seams, set out the development of the single sheet from which the junction could be made, showing the bends.

3. A cone and a cylinder intersect as in fig. 284. Draw plan and elevation to the sizes given. Determine the curve of intersection on elevation. Develop the cylindrical portion. Project a new elevation on X′Y′.

4. Fig. 285 shows a portion of curved roof from which a square chimney 3 ft. across the corners projects. Draw a side elevation showing the intersection and develop the projecting portion of the chimney. Scale 1 in. to 1 ft.

5. Fig. 286 shows a portion of a ventilator with a square base and octagonal shaft. Complete the elevation and develop the portion " A ", neglecting laps. Scale $\frac{3}{4}$ in. to 1 ft.

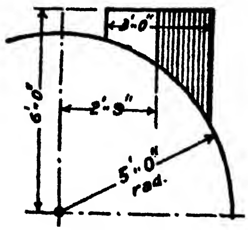


Fig. 285

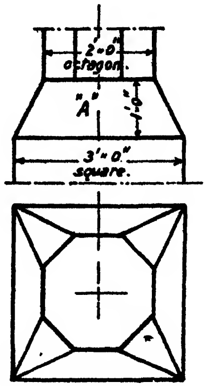


Fig. 286

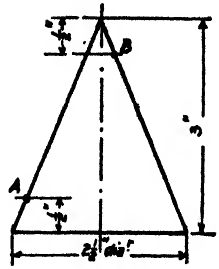


Fig. 287

6. Two points A and B on opposite sides of a cone are shown in fig. 287. Draw in elevation and plan the line which represents the shortest path on the surface between A and B. (*Note.*—The shortest distance between two points on a developable surface is that which is a straight line on the development.)

CHAPTER XVIII

HELICES AND SCREW THREADS

A **helix** is the path or locus traced out by a point which combines uniform rotation at constant radius round an axis with uniform motion parallel to the axis. The curve is important as being the basis of all screw threads.

The distance the point travels parallel to the axis during one complete revolution round it is called the **pitch.**

To Set Out a Helix (fig. 288).—Given the diameter and pitch, draw the plan and elevation of a cylinder having a diameter equal to that of the helix and on the elevation set up the pitch. Divide the circumference on plan and the pitch on elevation each into the same number of equal parts (12 will be found a convenient number and is used in the figure). For each successive rotation through one division on plan the tracing point will move through one division vertically on the elevation. Draw horizontal lines on the elevation to intersect projectors from the corresponding points on plan, and join these points of intersection by a curve to obtain the elevation of the helix.

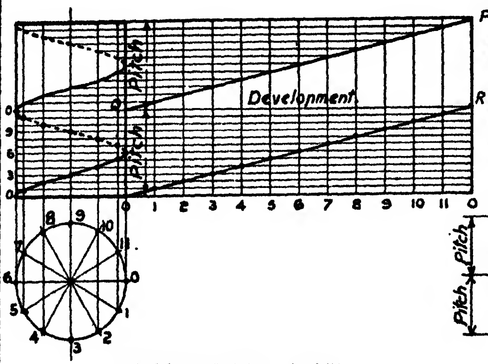
The development of the curve is a straight line as shown in the figure, and the angle PQR is termed the angle of the helix.

Screw Threads.—Screw threads are distinguished by names according to the profile of the section of the thread; thus triangular or V threads are illustrated in the Sellers thread (fig. 47) and the Whitworth thread (fig. 86), Square thread in fig. 290, and Buttress thread in fig. 298.

A screw thread is **right-handed** when it has to be turned in a clockwise direction to enter its nut, and **left-handed** when it has to be turned contra-clockwise.

Unless otherwise stated, standard threads are understood to be right-handed and are single threaded, i.e. the screw is composed of one continuous thread.

When screws are required in which the pitch is large compared with the diameter, in order to avoid the weakness caused by the large reduction of diameter at the bottom of the thread, screws are made with multiple



CONSTRUCTION OF HELIX

Fig. 288

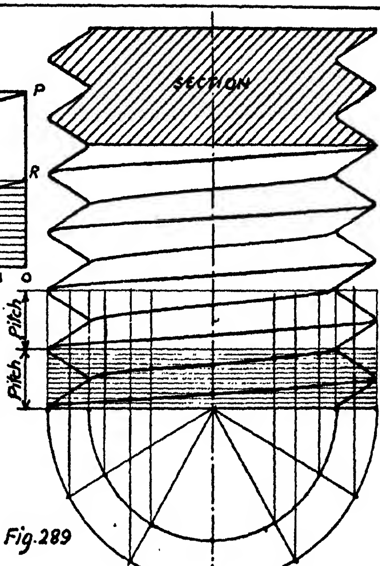


Fig. 289

SINGLE V. THREAD. R. HAND

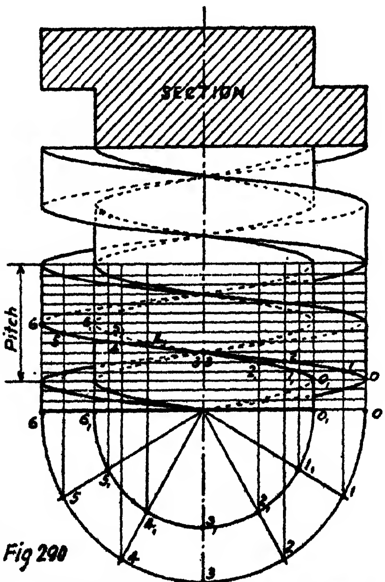


Fig 290

SINGLE SQUARE THREAD. L. HAND

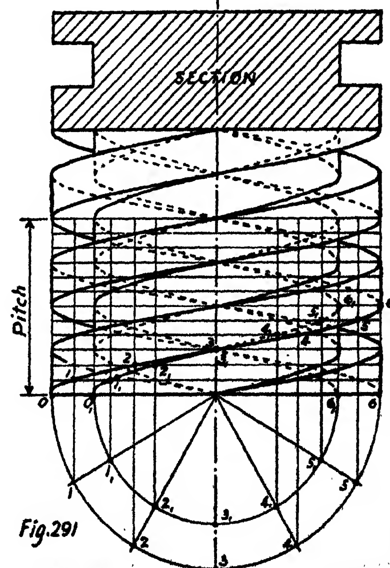


Fig. 291

DOUBLE SQUARE THREAD. R. HAND

threads, i.e. double threaded, treble threaded, &c. A double thread is shown in fig. 291.

In the examples of screw threads shown in figs. 289 to 291 the method of setting out is shown in each case, the helical curves being set out by the construction previously described. Work and space are saved by drawing only half plans of the circles representing the tops and bottoms of threads.

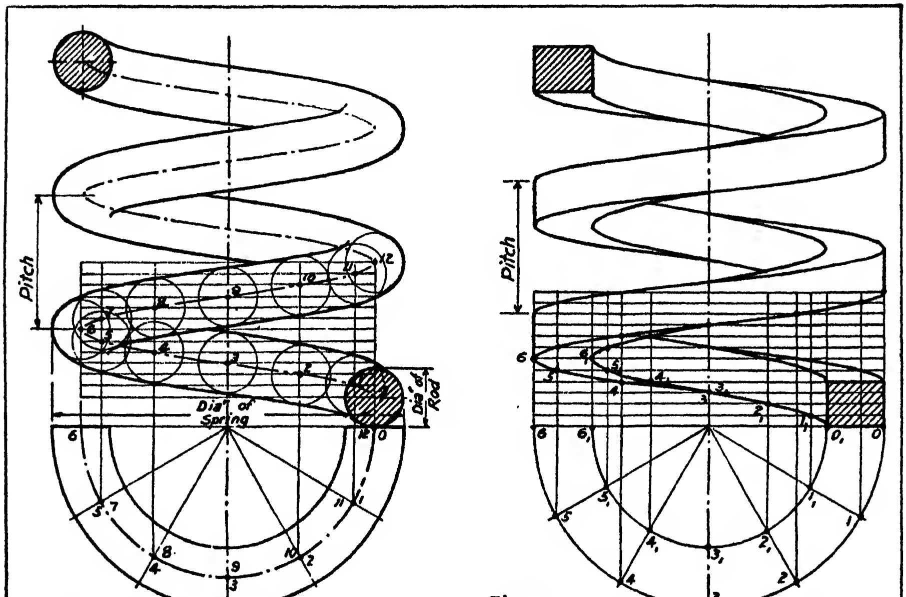


Fig. 292 ROUND SPRING

Fig. 293 RECTANGULAR SPRING

Spiral or Helical Springs.—For springs of square or rectangular section the method of setting out is similar (fig. 293).

For a spring of circular section (fig. 292) set out the helix representing the centre of the rod of which the spring is formed. A sphere equal in diameter to the round rod which moved with its centre on this helix would sweep out the surface of the spring. On the elevation, therefore, draw circles to represent positions of such a sphere and draw curves to touch these.

On working drawings where the pitch is small compared with the diameter, or where the scale is small, it is permissible to represent a thread by straight lines as shown in fig. 294, and for small details alternative stan-

dard conventions for representing threads are given in figs. 295 and 296. B.S.W. is a recognized standard abbreviation for British Standard Whitworth thread. L.H. indicates that a thread is left-handed, and unless so marked or otherwise stated, it is always assumed that threads are right-handed.

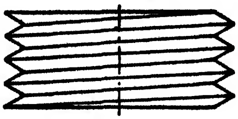


Fig. 294

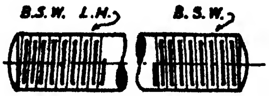


Fig. 295

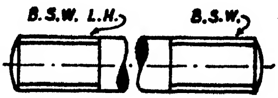


Fig. 296

EXERCISES

1. A line OP, $1\frac{3}{4}$ in. long, rotates horizontally at uniform speed about a vertical axis with its end O on the axis. At the same time the line OP travels up the axis at uniform speed a distance of $1\frac{1}{2}$ in. in each revolution. Draw plan and elevation of the path of the end P during 3 revolutions, and also of points Q and R on OP at distances of $\frac{1}{2}$ in. and 1 in. respectively from the end P.

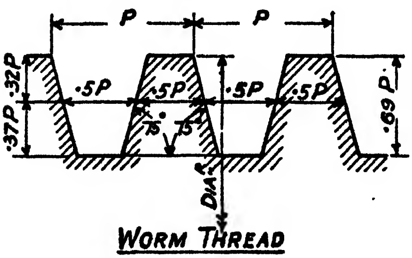


Fig. 297

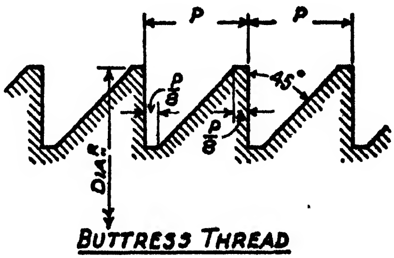


Fig. 298

2. Draw plan and elevation of a single-screw thread on a worm, the outside diameter being $4\frac{1}{2}$ in. and the pitch $1\frac{1}{2}$ in. Use the proportions of the thread given in fig. 297, which are stated in terms of the pitch P.

3. Draw plan and elevation of a single-threaded buttress screw, the outside diameter being $3\frac{1}{2}$ in. and the pitch 1 in. The proportions of the thread are given in terms of the pitch P in fig. 298.

CHAPTER XIX

LOCI

The locus of a point is the path which it traces out when it moves in conformity with definite conditions.

The name may be new, but the idea will already be familiar from the following cases among others which have previously been dealt with.

The locus of a point which moves so that it is always:—

At a **constant distance** from a **given straight line** is a **parallel straight line;**

Equidistant from **two intersecting lines** is the **bisector of the angle** between the lines;

Equidistant from two given points is the **perpendicular bisector** of the line joining the two points;

At at **constant distance** from a **given point** is a **circle;**

Equidistant from a given **straight line** and a **point** is a **parabola;**

At such distances from **two given points** that the **sum of those distances is constant** is an **ellipse.**

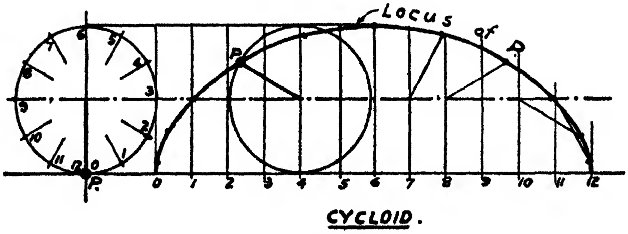


Fig. 299

There are many other loci of importance in drawing which are produced by points moving under defined conditions or restraints, a few examples of which are given here.

A Hyperbola is the locus of a point which moves so that the difference of its distances from two fixed points is constant.

A Cycloid is the locus of a point on the circumference of a circle which rolls on a straight line (fig. 299). This is of course the curve described by a point on the tread of a wheel rolling on a straight path.

The centre of the wheel describes a straight line. An intermediate point on a radius or spoke such as R or a point such as S on the extension of a radius would describe curves as shown in fig. 300.

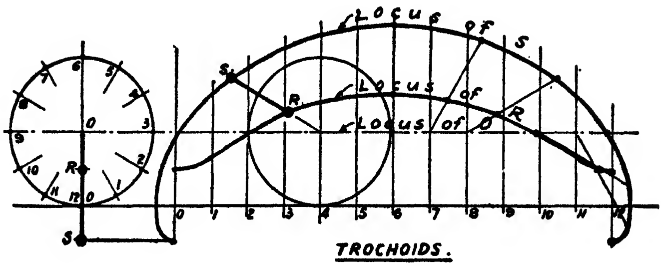


Fig. 300

To set out the curve, draw the rolling circle on tracing paper, dividing the circumference into intervals and marking the position of the tracing point whose locus is required.

Draw a line and on it step out intervals equal to those round the rolling

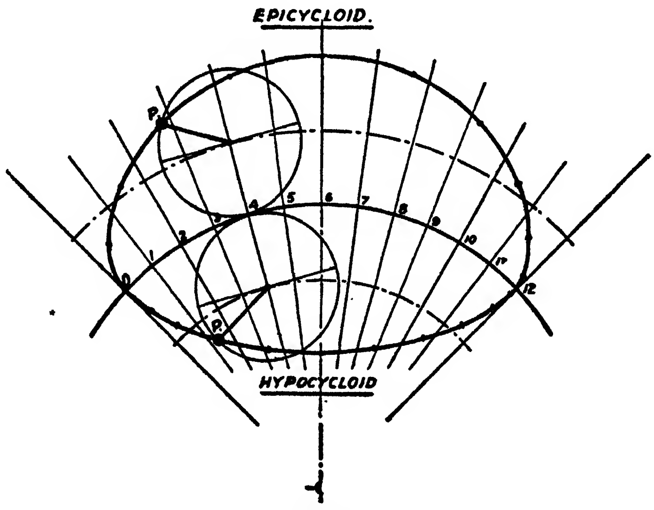


Fig. 301

circle. Draw perpendiculars through those points of division. Place the rolling circle in successive positions and prick through the position of the tracing point P, R, or S as the case may be and join by a fair curve.

The locus of a point on the circumference of a circle which rolls on the outer (convex) side of a fixed circle is an epicycloid (fig. 301).

If the rolling circle rolls on the **inner** (concave) **side** of the fixed circle, the locus of a point on the circumference is a **hypocycloid** (fig. 301).

Step out on the fixed circle the intervals into which the rolling circle is divided and draw radii. Apply the tracing of the rolling circle in successive positions on these radii, prick through the point P, and join by a fair curve.

The cycloidal curves are of importance as forming the basis of various shapes of teeth for toothed racks and gear wheels.

The locus of a point which travels uniformly along a line while the line rotates uniformly about one end is an **Archimedean Spiral** (fig. 302).

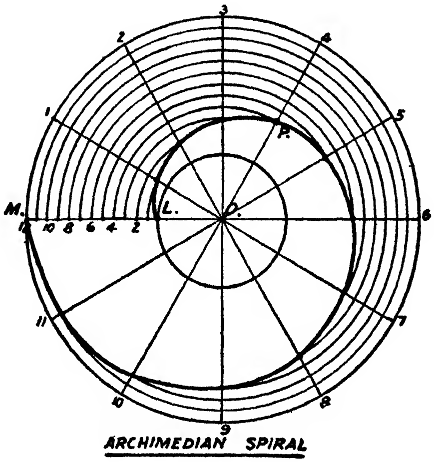


Fig. 302

Thus, if a point travels from L to M radially while the radial line MO completes one revolution, divide LM and the circumference into an equal number of parts and number the divisions as indicated. Where the circle through point 4 in LM meets the radius through point 4 on the circumference is a point on the spiral. Repeat for other points and join by a fair curve.

A line moves so that **one end rotates in a circle** while the **other end slides on a line** which passes through the centre of the circle (fig. 303). To draw the locus of any point on the line.

This will be recognized as the familiar case of the **crank and connecting-rod.** The crank rotates in the circle with ON as radius, the crosshead end sliding on KL, and the locus of P is required.

Trace the line MN on tracing paper and mark the point P. Apply the tracing in successive positions with the ends on the crank circle and

KL respectively, prick through the positions of P and join these to obtain the curve.

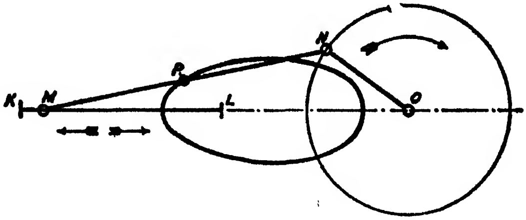


Fig. 303

Fig. 304 represents a link motion consisting of two rocking arms LM and NO centred at L and O and connected by a link MN. The locus of a point P on the connecting link is required.

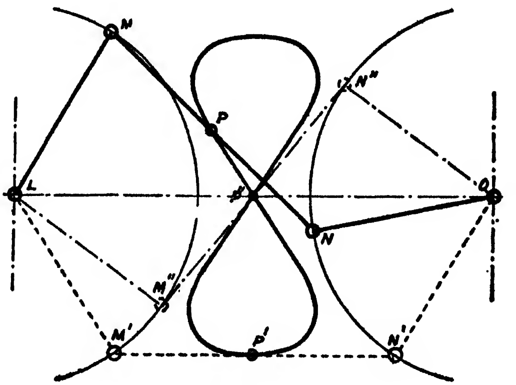


Fig. 304

Draw circles representing the paths of M and N. Trace the link MN and mark the point P. Apply the tracing in successive positions such as M′N′ and M″N″ and prick through the positions of P.

EXERCISES

1. A rod 4 in. long moves so that one end travels vertically down a wall while the other end travels horizontally on the floor at right angles to the wall. Trace the loci of points 1 in., 2 in., and 3 in. from the end of the rod. (Note that the mid-point describes a quarter of a circle, the other two points quarter-ellipses.)

2. Draw a cycloid, the rolling circle being $1\frac{1}{2}$ in. diameter.

3. A circle $1\frac{1}{2}$ in. diameter rolls inside a circle $4\frac{1}{2}$ in. diameter. Draw the locus of a point on the circumference of the rolling circle and also that of a point 1 in. from its centre for a complete circuit of the fixed circle.

4. A circle $1\frac{1}{2}$ in. diameter rolls inside a circle 3 in. diameter. Draw the locus of a point on the circumference. (Note that this locus is a **diameter of the fixed circle.** This particular case of the hypocycloid is used to produce a straight-line motion.)

5. Draw the locus of a point which moves so that it is always equidistant from the circumference of a fixed circle $2\frac{1}{2}$ in. diameter and a fixed straight line 2 in. from the centre of the circle.

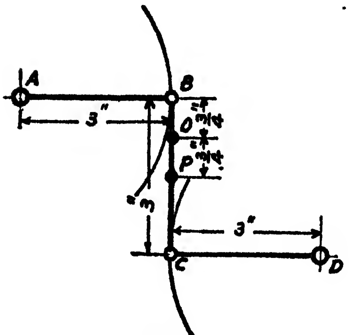


Fig. 305

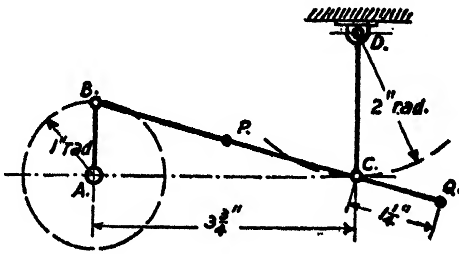


Fig. 306

6. Draw the locus of a point which is always equidistant from the circumferences of two fixed circles 1 in. and 3 in. diameter respectively, the centres of which are $2\frac{1}{2}$ in. apart.

7. Draw an equilateral triangle ABC, 3 in. sides. With A, B, and C respectively as centres, draw circles $\frac{3}{4}$ in., 1 in., and $1\frac{1}{4}$ in. radius. By the use of two intersecting loci similar to that in Exercise 6, find the centre of, and draw the internal circle which touches the three given circles.

8. A crank is 18 in. radius and a connecting-rod 4 ft. 6 in. long. Draw the loci of points 1 ft. 6 in. from each end of the connecting-rod. Scale 1 in. to 1 ft.

9. Fig. 305 represents a link motion. AB and CD rotate about A and D respectively. Set out the loci of the points O and P.

10. Fig. 306 represents a link motion. The crank AB rotates about A. The rocking lever CD is centred at D. Set out the locus of P, the centre point of the connecting link BC, and of the point Q on the extension of BC.

CHAPTER XX

LINES AND PLANES

It will already have been seen that when a line is parallel to one of the planes of projection, its projection on that plane displays its true length. It also displays the true angle between the line and the other plane of projection to which it is inclined.

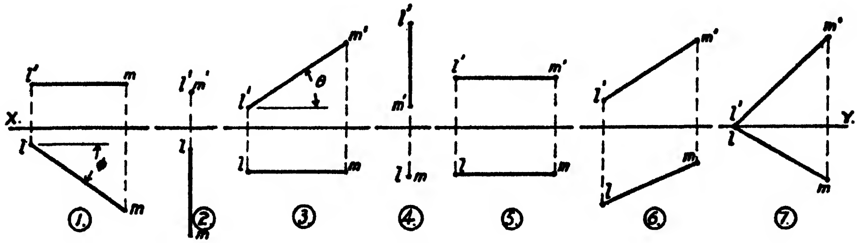


Fig. 307

Thus the lines shown in fig. 307, cases 1 and 2, are parallel to H.P., and the true lengths of LM are displayed in plan; cases 3 and 4 are parallel to V.P., and the true lengths are displayed in elevation. The angles θ and ϕ display the true inclinations to H.P. and V.P. In case 5 the line is parallel to both H.P. and V.P., and the true length seen in both plan and elevation.

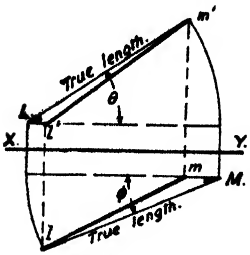


Fig. 308

In cases 6 and 7 the line LM is inclined to both planes of projection, and neither in plan nor elevation is the true length apparent, nor the true inclination to H.P. or V.P.

To Find the True Length of a Line and its Inclination to H.P. or V.P. (fig. 308).—Imagine the line LM to rotate about a vertical projector, without altering its inclination, until it occupies a position parallel to V.P. In this position the true length and inclination to H.P. will be seen in elevation.

Thus in fig. 308 LM is rotated about the vertical projector through M until its plan is parallel with XY. The elevation Lm' in this new position gives the true length and θ the true inclination to H.P.

Similarly, had the line been rotated about the horizontal projector through L into a new position with the elevation parallel to XY, the plan lM would show the true length and ϕ the true inclination to V.P.

Another method sometimes more convenient is shown in fig. 309. In the plan of the line draw perpendiculars through the ends l and m. Make lL and mM respectively equal to the heights of L and M above XY as obtained from the elevation. LM will then be the true length, and θ, the angle it makes with lm, the true inclination to H.P.

Similarly, the angle ϕ would be obtained by erecting perpendiculars l'L, m'M, and making these equal to the distances of l and m from XY in

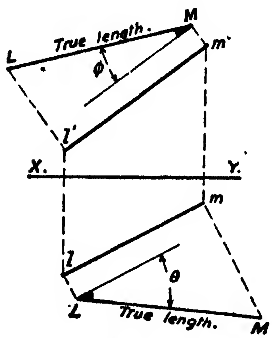


Fig. 309

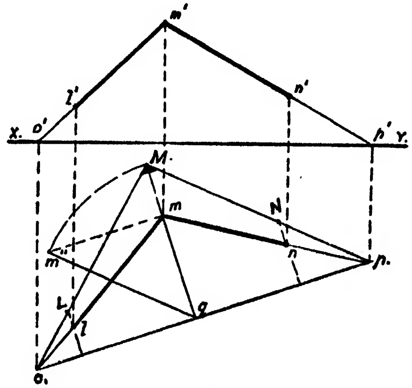


Fig. 310

plan. LM is again the true length, and the angle ϕ between this and $l'm'$ the true inclination to V.P.

Given the Projections of two Intersecting Lines, to Find the True Angle between Them (fig. 310).—The lines are LM and MN, the angle LMN being required. Produce the lines to meet H.P., in other words find their **horizontal traces.** In the projections this is done by producing $m'l'$ to meet XY in o' and projecting this to meet ml produced in plan at o. O is the horizontal trace of LM.

Similarly, find p and p'.

If the triangle OMP is rotated about OP as a "hinge" until M is in the H.P., the true angle at M will be displayed. Draw qm perpendicular to op. The true length and inclination of QM is found by setting up mm'' equal to the height of m' above XY. With q as centre and radius qm'', draw an arc to cut qm in M.

oMp is the required angle and is obviously equal to LMN.

Planes are represented in projections by the lines in which they intersect the planes of projection, the **vertical trace** (V.T.) and **horizontal trace** (H.T.) being respectively the lines in which a plane intersects the V.P. or H.P. Obviously these traces intersect on XY.

Planes inclined to **one** plane of projection but perpendicular to the other are **inclined planes** (fig. 311, 1 and 2). Such planes present an edgewise view either in elevation or plan and display their true inclination θ or ϕ as the case may be.

Planes inclined to **both** planes of projection are **oblique planes** (fig. 311, 3), and the angles which such planes make with the planes of projection are not apparent either in plan or elevation.

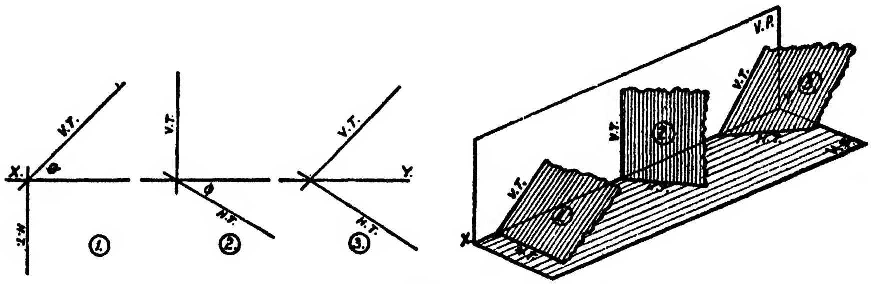


Fig. 311

The following facts must be verified and memorized:

A line lying in a plane and **parallel to H.P.** has its **plan parallel to H.T.**

A line lying in a plane and **parallel to V.P.** has its **elevation parallel to V.T.**

Any line **perpendicular to a plane** has its **projections perpendicular to the traces** of the plane, i.e. elevation and plan perpendicular to V.T. and H.T. respectively.

The **angle between two planes** may be thus defined: If from a point on the intersection of two planes, lines perpendicular to the intersection are drawn in the planes, the angle between these two lines is equal to the angle between the planes.

Given the Traces of an Oblique Plane, to Find the True Inclination to H.P. and V.P. (fig. 312).—Select any point *m* in H.T. Through this imagine lines drawn at right angles to H.T., one in the plane and one in H.P. The angle between these will be the true inclination to H.P.

Through *m* in plan draw perpendicular to H.T. to meet XY. Project the elevations *m'l'* and *m'n'*. Rotate the triangle LMN about the projector

LM until it is in the V.P., i.e. until the plan is in XY as shown. The angle θ at M will be the true inclination to H.P.

Similar procedure, selecting a point in V.T., is illustrated on the right of the figure to obtain the angle ϕ between the plane and V.P.

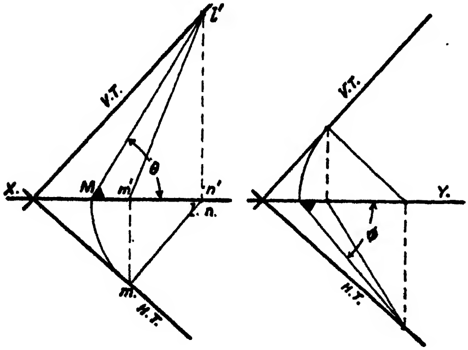


Fig. 312

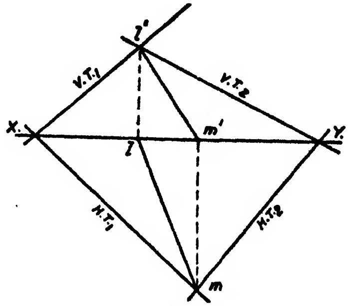


Fig. 313

To Determine the Line in which Two Given Oblique Planes Intersect Each Other (fig. 313).—The point l', where the two vertical traces intersect, is obviously on the required intersection, as is also the

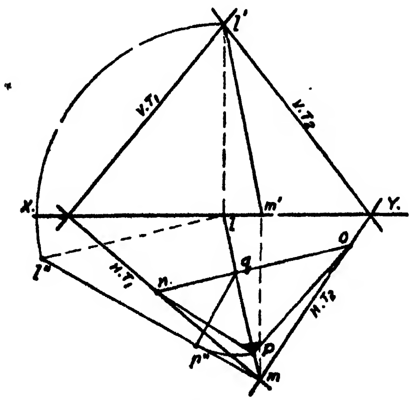


Fig. 314

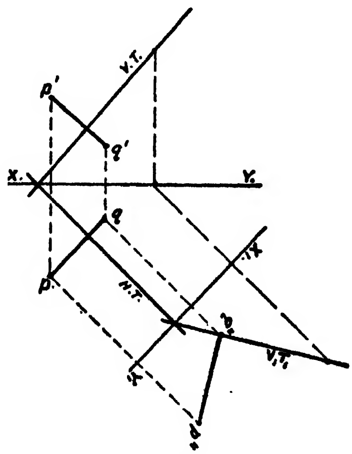


Fig. 315

point *m*, where the two horizontal traces intersect. LM is therefore the required intersection and its projections are *l'm'* and *lm*.

To Determine the Angle between Two Intersecting Planes whose Traces are Given (fig. 314).—Draw the projections of their intersection LM.

Perpendicular to *lm* in plan draw any line *no*, cutting HT_1 and HT_2 at *n* and *o* and *lm* at *q*.

Rotate LM into the H.P. at $l''m$. From *q* draw qp'' perpendicular to $l''m$.

The triangle OPN is a section of the two planes perpendicular to their intersection LM.

With *q* as centre and radius qp'', draw an arc to cut LM at P. The angle *oPn* will be the required angle between the planes.

To Draw a Perpendicular to a Plane from a Given Point P outside the Plane (fig. 315).—Through *p* and *p'* draw lines perpendicular to H.T. and V.T. These will represent projections of the required line. To find the point where the line meets the plane, take an auxiliary end view on X_1Y_1 at right angles to H.T. The plane will then be represented by V_1T_1 and the point by p''. From p'' draw a perpendicular to V_1T_1 to meet it at q''. Project q'' back at *q* and *q'* on the plan and elevation. PQ will be the required perpendicular.

EXERCISES

1. The plan of a line is 2 in. long, and one end of the line is 1 in. higher than the other. What is its true length?

2. The plan of a line is 2½ in. long, and its true length 4 in. How much higher is one end than the other?

3. The plan of a line is 2 in. long, and its true length 4 in. What angle does the line make with the horizontal?

4. The plan of a line is 3 in. long, and the line is inclined at 45° to the H.P. What is the true length?

5. A mast 30 ft. high is stayed by means of four equal guy ropes, the lower ends of which are fastened to four points on the ground at the corners of a square of 20 ft. side. What is the length of a guy rope?

6. A point in a room is situated 3 ft. above the floor, and 4 ft. from a wall. What is its distance from the edge in which the floor and wall meet?

7. A point in a room is 4 ft. above the floor, and 3 ft. 6 in. and 5 ft. 6 in. respectively from two adjoining walls. What is its distance from the corner where the walls and floor meet?

Set this up full size in a corner of the room to check your results.

8. A box measures 3 ft. × 4 ft. × 12 ft. What is the greatest overall dimension, i.e. the length of a diagonal of the box?

9. A hipped roof (fig. 316) for a building 35 ft. long × 20 ft. wide has a vertical rise of 5 ft. 6 in. from eaves to ridge. Draw to $\frac{1}{8}$ in. to 1 ft. and determine:

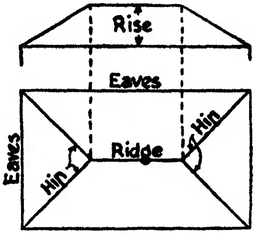


Fig. 316

(*a*) The actual length measured along the hip;

(*b*) The length from eaves to ridge (measured perpendicularly between them);

(*c*) The length measured along the ridge;

(*d*) The angle the hip makes with the horizontal;

(*e*) The true shape of the triangle formed by the end eaves and two hips;

(*f*) The true shape of the quadrilateral formed by the side eaves, ridge, and two hips.

10. A stick 6 ft. long has one end on the floor and the other against a wall. The lower end is 2 ft. 0 in. from the wall, and the upper end 2 ft. 6 in. above the floor. What angle does the stick make with the floor and wall respectively? Try this actually with a stick 6 ft. long and so check your drawing.

11. A tripod stand has legs 5 ft. long. The feet are spread out on level ground to rest at the corners of an equilateral triangle 3 ft. 6 in. side. What is the height of the apex of the tripod above the ground?

12. A tripod has 3 equal legs 8 ft. long. The feet are spread out on level ground to rest at the corners of at triangle having sides of 5 ft., 6 ft., and 7 ft. respectively. Draw a plan and elevation of the tripod and measure the height of the apex. Scale $\frac{1}{2}$ in. = 1 ft.

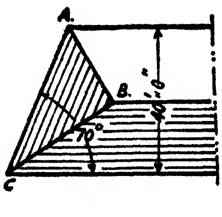


Fig. 317

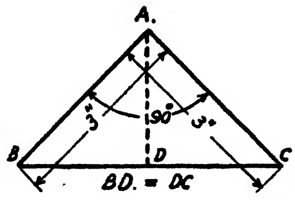


Fig. 318

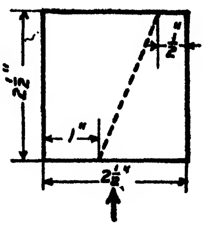


Fig. 319

13. The plan of the hipped end of a roof is given (fig. 317). The side slopes and hipped end are all inclined at 30°. Draw to a scale of 10 ft. to 1 in. and set out the length of the hips AB and BC. Measure the angles which the hips make with the horizontal.

14. Fig. 318 represents a sheet of thin metal. It is bent along the line AD until the faces ABD and ACD are at right angles. Draw a plan and

elevation when the bent sheet rests with the edges AB and AC on a horizontal surface.

15. Fig. 319 represents a sheet of thin metal. It is bent along the dotted line until the angle between the two portions is 60°. Draw a plan when so bent and project an elevation looking in the direction of the arrow.

MISCELLANEOUS EXERCISES

1. Fig. 320 shows a double-ended spanner. The widths between jaws suitable for the nuts of $\frac{3}{4}$ in., $\frac{7}{8}$ in., 1 in., and $1\frac{1}{8}$ in. bolts are $1\frac{5}{16}$ in., $1\frac{1}{2}$ in., $1\frac{11}{16}$ in., and $1\frac{7}{8}$ in. respectively. Draw $\frac{1}{4}$ full size a similar spanner suitable for $\frac{3}{4}$ in. nuts one end and $\frac{7}{8}$ in. the other, overall length $13\frac{1}{2}$ in.; also one suitable for 1 in. at one end and $1\frac{1}{8}$ in. the other, overall length 18 in.

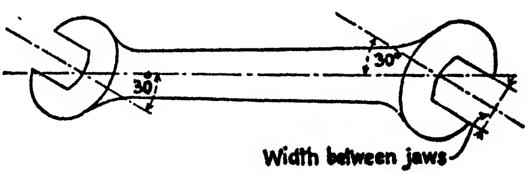


Fig. 320

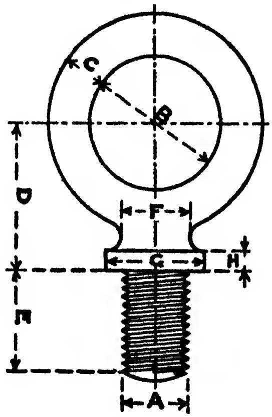


Fig. 321

2. Fig. 321 represents a screwed ringbolt. Three sizes are given in the following list:

A	B	C	D	E	F	G	H
$\frac{3}{4}''$	$1\frac{1}{8}''$	$\frac{9}{16}''$	$1\frac{3}{4}''$	$1\frac{1}{4}''$	$\frac{3}{4}''$	$1\frac{1}{4}''$	$\frac{1}{4}''$
$1''$	$2''$	$\frac{5}{8}''$	$2\frac{3}{16}''$	$1\frac{1}{2}''$	$1\frac{1}{16}''$	$1\frac{1}{2}''$	$\frac{5}{16}''$
$1\frac{1}{4}''$	$2\frac{3}{4}''$	$\frac{7}{8}''$	$3''$	$2''$	$1\frac{5}{8}''$	$2\frac{1}{4}''$	$\frac{1}{2}''$

Make a full-size drawing of each size.

3. In cutting rectangular beams from circular logs the section of the strongest form that can be cut from a given log is obtained by the method indicated in fig. 322. The diameter AC is divided into four, and perpendiculars EB and FD drawn through E and F. The rectangle ABCD gives the proportions of the beam which, of all those which could be cut from

the given log, would sustain the greatest load before failure. Determine and write down the dimensions of the strongest beams which can be cut from logs 16 in. and 18 in. diameter respectively.

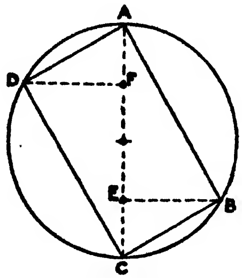


Fig. 322

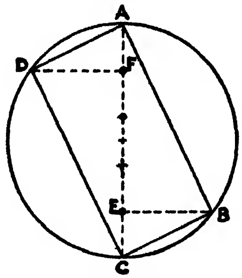


Fig. 323

4. The section of the **stiffest** beam that can be cut from a circular log is obtained by the method indicated in fig. 323, the diameter in this case being divided into five, the rectangle ABCD giving the proportions of the beam which, of all those which could be cut from the given log, would deflect or sag least under a given load. Determine and write down the dimensions of the stiffest beams which can be cut from logs 16 in. and 18 in. diameter respectively.

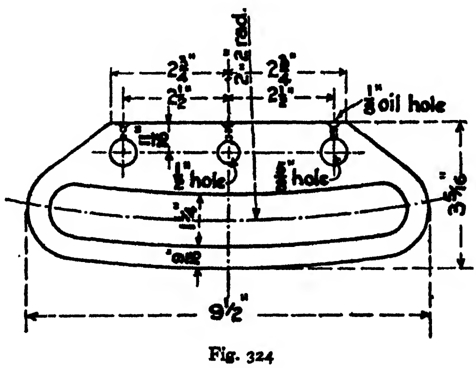


Fig. 324

5. Fig. 324 represents a link plate for the valve motion of a small steam engine. Draw this ½ full size and add a vertical section at the centre, the plate being 1⅛ in. thick.

6. Fig. 325 gives two views of a crane hook suitable for a 3-ton crane. Draw these to a scale of ½ full size.

7. Fig. 326 shows the proportions commonly used for the corrugations of galvanized corrugated roofing sheets. The most usual sizes for these are

3 in. and 5 in. pitch, i.e. the distance P from centre to centre of corrugations is 3 in. or 5 in. Draw ½ full size two complete " flutes " as shown for each of these sizes.

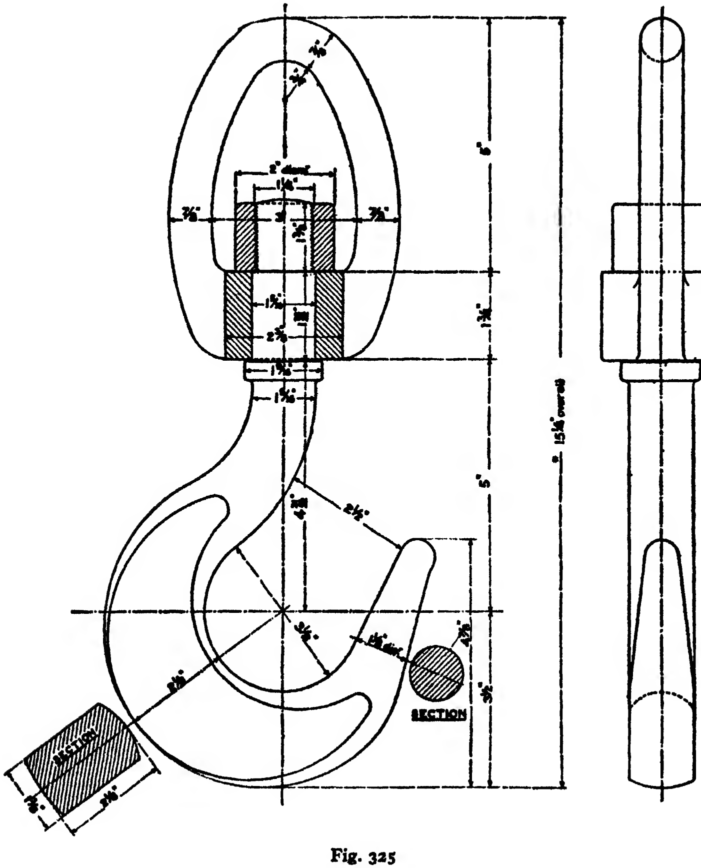


Fig. 325

8. Fig. 327 is a section of an earth embankment. A road 20 ft. wide is to be cut through this at the level AB and at right angles to the embankment on plan, the earth slopes at the side of the road making an angle of

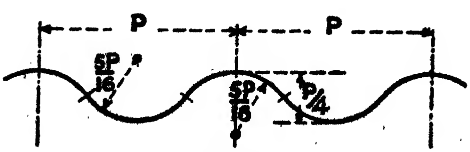


Fig. 326

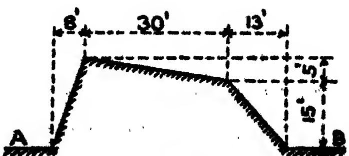


Fig. 327

60° with the horizontal. Draw a plan of the embankment showing the road and the outline of the sloping cutting. Scale 20 ft. to 1 in.

9. Fig. 328 shows the forms of rivet " heads " and " points " in common use, dimensions being given in terms of D, the diameter of the shank of the rivet in each case. The shapes illustrated are known as:

(*a*) Snap head and snap point.
(*b*) Pan head and snap point.
(*c*) Snap head and countersunk point.
(*d*) Rounded countersunk head and snap point.
(*e*) Snap head and conical point.

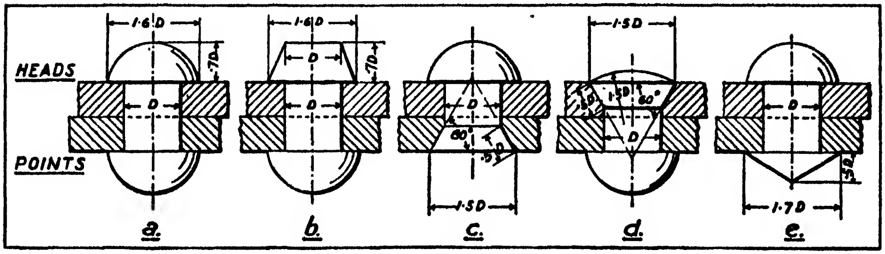


Fig. 328

Set these out full size for rivets ¾ in., ⅞ in., and 1 in. diameter respectively, showing them applied to clinch three thicknesses of ½ in. plate.

10. The transverse vertical section of a compound girder is given in fig. 329. Draw full size a section compounded of two flange plates 12 in. × ¾ in., and a rolled-steel joist or beam 12 in. × 6 in. (fig. 79 and table, p. 36), adding ⅞ in. diameter rivets of the form shown at *a*, fig. 328.

11. The transverse vertical section of a mild-steel built-up plate girder is given in fig. 330. Draw ½ full size a section of a girder built up of two flange plates 13 in. × ½ in., a web plate 18 in. × ⅜ in., and four angles each 3½ in. × 3½ in. × ½ in. (fig. 75 and table, p. 35). Rivets ¾ in. diameter of the form shown at *a*, fig. 328.

12. Fig. 331 gives the elevation of a dressed granite cap stone for a masonry pier. Draw to a scale of ⅛ full size the given view, and add a side elevation and plan of the cap, given that the masonry pier is 2 ft. 6 in. × 2 ft. 9 in.

13. Draw an isometric projection of the cap stone shown in the foregoing example. Scale ⅛ full size.

14. Fig. 332 gives the plan and elevation of the line diagram of a braced tower. Draw to a scale of ¼ in. to 1 ft. Find the actual length of each of

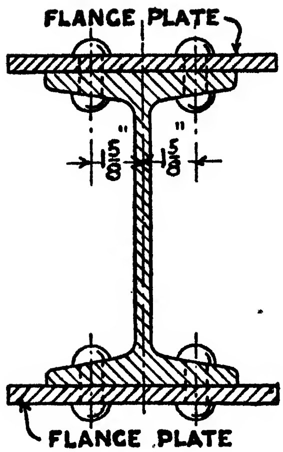


Fig. 329

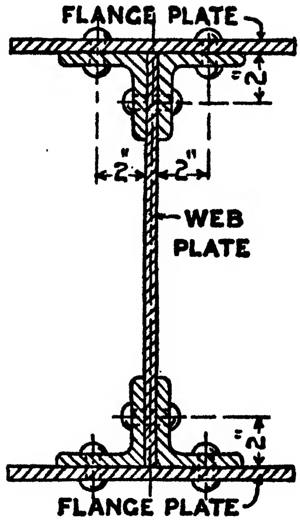


Fig. 330

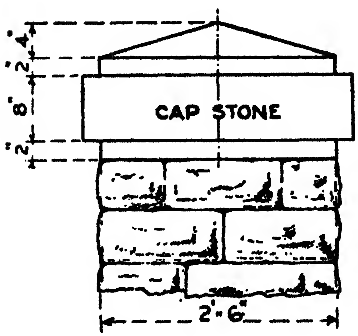


Fig. 331

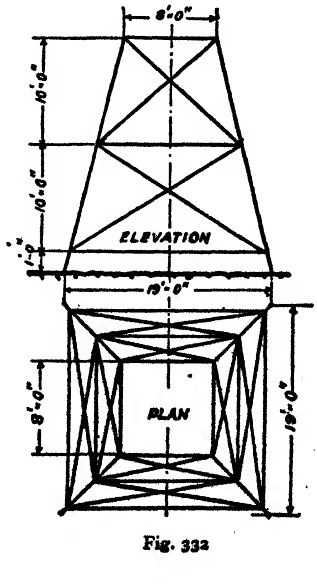


Fig. 332

the four columns at the corners and of the diagonal bracings, these lengths not being apparent from either of the views given.

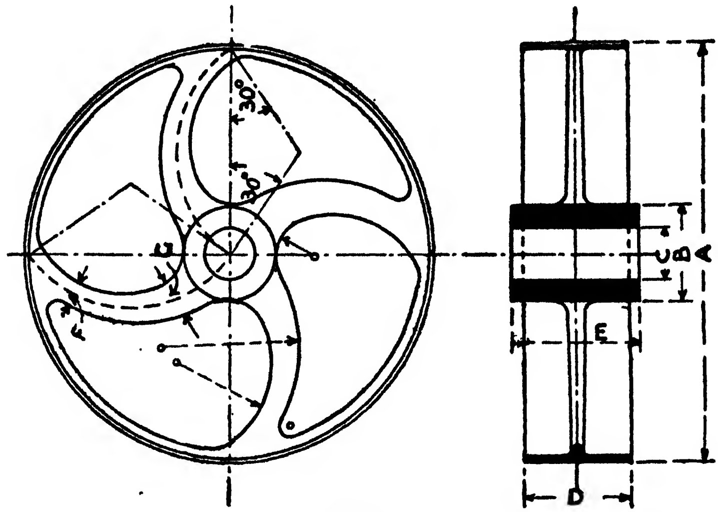


Fig. 333

15. Fig. 333 shows an elevation and central cross-section of a cast-iron pulley, the dimensions of which are as follows:

A—outside diameter	=	30 in.
B—diameter of boss	=	7 in.
C—diameter of shaft	=	$3\frac{1}{2}$ in.
D—width of " face " of pulley	=	8 in.
E—length of boss	=	10 in.
F—width of tapered arm	=	$2\frac{1}{2}$ in.
G—width of tapered arm	=	$3\frac{1}{2}$ in.
Thickness of rim	=	$\frac{3}{8}$ in.

Draw these views to a scale of $1\frac{1}{2}$ in. to 1 ft.

NUMBERS, SQUARES, CUBES, SQUARE ROOTS AND CUBE ROOTS

Number.	Square.	Cube.	Square Root.	Cube Root.
1	1	1	1.000	1.000
2	4	8	1.414	1.259
3	9	27	1.732	1.442
4	16	64	2.000	1.587
5	25	125	2.236	1.709
6	36	216	2.449	1.817
7	49	343	2.645	1.912
8	64	512	2.828	2.000
9	81	729	3.000	2.080
10	100	1,000	3.162	2.154
11	121	1,331	3.316	2.223
12	144	1,728	3.464	2.289
13	169	2,197	3.605	2.351
14	196	2,744	3.741	2.410
15	225	3,375	3.872	2.466
16	256	4,096	4.000	2.519
17	289	4,913	4.123	2.571
18	324	5,832	4.242	2.620
19	361	6,859	4.358	2.668
20	400	8,000	4.472	2.714
21	441	9,261	4.582	2.758
22	484	10,648	4.690	2.802
23	529	12,167	4.795	2.843
24	576	13,824	4.898	2.884
25	625	15,625	5.000	2.924
26	676	17,576	5.099	2.962
27	729	19,683	5.196	3.000
28	784	21,952	5.291	3.036
29	841	24,389	5.385	3.072
30	900	27,000	5.477	3.107
31	961	29,791	5.567	3.141
32	1,024	32,768	5.656	3.174
33	1,089	35,937	5.744	3.207

Number.	Square.	Cube.	Square Root.	Cube Root.
34	... 1,156	 39,304	5.830	3.239
35	1,225	42,875	5.916	3.271
36	... 1,296	 46,656	6.000	3.301
37	1,369	50,653	6.082	3.332
38	... 1,444	 54,872	6.164	3.361
39	1,521	59,319	6.244	3.391
40	... 1,600	 64,000	6.324	3.419
41	1,681	68,921	6.403	3.448
42	... 1,764	 74,088	6.480	3.476
43	1,849	79,507	6.557	3.503
44	... 1,936	 85,184	6.633	3.530
45	2,025	91,125	6.708	3.556
46	... 2,116	 97,336	6.782	3.583
47	2,209	103,823	6.855	3.608
48	... 2,304	 110,592	6.928	3.634
49	2,401	117,649	7.000	3.659
50	... 2,500	 125,000	7.071	3.684
51	2,601	132,651	7.141	3.708
52	... 2,704	 140,608	7.211	3.732
53	2,809	148,877	7.280	3.756
54	... 2,916	 157,464	7.348	3.779
55	3,025	166,375	7.416	3.802
56	... 3,136	 175,616	7.483	3.825
57	3,249	185,193	7.549	3.848
58	... 3,364	 195,112	7.615	3.870
59	3,481	205,379	7.681	3.892
60	... 3,600	 216,000	7.745	3.914
61	3,721	226,981	7.810	3.936
62	... 3,844	 238,328	7.874	3.957
63	3,969	250,047	7.937	3.979
64	... 4,096	 262,144	8.000	4.000
65	4,225	274,625	8.062	4.020
66	... 4,356	 287,496	8.124	4.041
67	4,489	300,763	8.185	4.061
68	... 4,624	 314,432	8.246	4.081
69	4,761	328,509	8.306	4.101
70	... 4,900	 343,000	8.366	4.121
71	5,041	357,911	8.426	4.140
72	... 5,184	 373,248	8.485	4.160
73	5,329	389,017	8.544	4.179
74	... 5,476	 405,224	8.602	4.198

Number.	Square.	Cube.	Square Root.	Cube Root.
75	5,625	421,875	8.660	4.217
76	5,776	438,976	8.717	4.235
77	5,929	456,533	8.744	4.254
78	6,084	474,552	8.831	4.272
79	6,241	493,039	8.888	4.290
80	6,400	512,000	8.944	4.308
81	6,561	521,441	9.000	4.326
82	6,724	551,368	9.055	4.344
83	6,889	571,787	9.110	4.362
84	7,056	592,704	9.165	4.379
85	7,225	614,125	9.219	4.396
86	7,396	636,056	9.273	4.414
87	7,569	658,503	9.327	4.431
88	7,744	681,472	9.380	4.447
89	7,921	704,969	9.433	4.461
90	8,100	729,000	9.486	4.481
91	8,281	753,571	9.539	4.497
92	8,464	778,688	9.591	4.514
93	8,649	804,357	9.643	4.530
94	8,836	830,584	9.695	4.546
95	9,025	857,375	9.746	4.562
96	9,216	884,736	9.797	4.578
97	9,409	912,673	9.848	4.594
98	9,604	941,192	9.899	4.610
99	9,801	970,299	9.949	4.626
100	10,000	1,000,000	10.000	4.641

DIAMETERS OF CIRCLES, CIRCUMFERENCES AND AREAS

Diameter.	Circumference.	Circular Area.	Diameter.	Circumference.	Circular Area.
1	3.1416	0.7854	34	106.81	907.92
2	6.28	3.14	35	109.96	962.11
3	9.42	7.07	36	113.10	1017.88
4	12.57	12.57	37	116.24	1075.21
5	15.71	19.63	38	119.38	1134.11
6	18.85	28.27	39	122.52	1194.59
7	21.99	38.48	40	125.66	1256.64
8	25.13	50.27	41	128.80	1320.25
9	28.27	63.62	42	131.95	1385.44
10	31.42	78.54	43	135.09	1452.20
11	34.56	95.03	44	138.23	1520.53
12	37.70	113.10	45	141.37	1590.43
13	40.84	132.73	46	144.51	1661.90
14	43.98	153.94	47	147.65	1734.94
15	47.12	176.71	48	150.80	1809.56
16	50.26	201.06	49	153.94	1885.74
17	53.41	226.98	50	157.08	1963.50
18	56.55	254.47	51	160.22	2042.82
19	59.69	283.53	52	163.36	2123.72
20	62.83	314.16	53	166.50	2206.18
21	65.97	346.36	54	169.65	2290.22
22	69.11	380.13	55	172.79	2375.83
23	72.26	415.48	56	175.93	2463.01
24	75.40	452.39	57	179.07	2551.76
25	78.54	490.87	58	182.21	2642.08
26	81.68	530.93	59	185.35	2733.97
27	84.82	572.56	60	188.50	2827.43
28	87.96	615.75	61	191.64	2922.47
29	91.11	660.52	62	194.78	3019.07
30	94.25	706.86	63	197.92	3117.25
31	97.39	754.77	64	201.06	3216.99
32	100.53	804.25	65	204.20	3318.31
33	103.67	855.30	66	207.34	3421.19

Diameter.	Circumference.	Circular Area.	Diameter.	Circumference.	Circular Area.
67	210.49	3525.65	84	263.89	... 5541.77
68	213.63	... 3631.68	85	267.03	5674.50
69	216.77	3739.28	86	270.18	... 5808.80
70	219.91	... 3848.45	87	273.32	5944.68
71	223.05	3959.19	88	276.46	... 6082.12
72	226.19	... 4071.50	89	279.60	6221.14
73	229.34	4185.39	90	282.74	... 6361.73
74	232.48	... 4300.84	91	285.88	6503.88
75	235.62	4417.86	92	289.03	... 6647.61
76	238.76	... 4536.46	93	292.17	6792.91
77	241.90	4656.63	94	295.31	... 6939.78
78	245.04	... 4778.36	95	298.45	7088.22
79	248.19	4901.67	96	301.59	... 7238.23
80	251.33	... 5026.55	97	304.73	7389.81
81	254.47	5153.00	98	307.88	... 7542.96
82	257.61	... 5281.02	99	311.02	7697.69
83	260.75	5410.61	100	314.16	... 7853.98